FALSE VACUUM DECAY IN QUANTUM SPIN CHAINS

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False vacuum decay has been at the centre of physics research in cosmology and quantum field theory. As the timescales and the dynamics of this phenomenon are difficult to observe and describe analytically, there have been only limited options for its study. With advancements in condensed matter physics and quantum technologies the study of false vacuum decay is becoming more accessible. Metastable systems, with dynamics analogue to the false vacuum decay, can be observed on timescales that allow measurements. In particular, the transverse Ising model can present metastable decay in non-equilibrium dynamics following a quantum quench. Numerical simulations show vacuum decay on short timescales. This presents an opportunity for further study on longer timescales using quantum annealing devices.

RAZPAD LAŽNEGA VAKUUMA V KVANTNIH SPINSKIH VERIGAH

Razpad lažnega vakuuma je v ospredju raziskov na področju kozmologije in kvantne teorije polja. Časovne skale in dinamiko tega pojava je težko opazovati in opisati analitično, zato obstajajo samo omejene možnosti za njegov študij. Z napredki v fiziki kondenzirane snovi in kvantnih tehnologijah postaja raziskovanje razpada lažnega vakuuma bolj dostopno. Metastabilne sisteme, ki imajo dinamiko analogno razpadu lažnega vakuuma, lahko opazujemo na časovnih skalah, ki omogočajo meritve. Eden izmed takih sistemov je transverzalni Isingov model, v katerem se lahko pojavi razpad metastabilnega stanja v neravnovesni dinamiki, ki sledi nenadni menjavi smeri magnetnega polja. Numerične simulacije pokažejo razpad vakuuma na kratkih časovnih skalah, kar predstavlja priložnost za nadaljnji študij na daljših časovnih skalah z uporabo kvantih računalnikov.

1. Introduction

False vacuum decay is a central idea in quantum field theory and cosmology. It describes a scenario where a system gets 'stuck' in a metastable false vacuum state, as illustrated in figure 1, and transitions to the true vacuum state by bubble nucleation, which entails a spontaneous formation of bubbles of true vacuum via quantum tunneling and their subsequent rapid expansion that is driven by the energy difference between the two vacua. This phenomenon was first studied in [1] and [2]. Bubble formation is, however, not only relevant in cosmology but is the mechanism behind first-order classical and quantum phase transitions. This paper discusses how this fact can be used to study false vacuum decay in quantum spin chains. The cosmological aspects of vacuum instabillity are discussed first, followed by the manifestations of metastable systems in condensed matter, focusing on the timescales of the transitions. Lastly, the false vacuum decay in the transverse Ising model is analyzed.

2. False vacuum

Vacuum is defined as the state with the lowest possible energy. However, if this vacuum state is only the state that has the lowest energy locally, not globally, this is called a metastable vacuum. True vacuum is the state that has the lowest possible energy globally. In quantum field theory there can exist multiple local minima of the potential. Any of these minima represent the vacuum in the sense that we can expand the fields around their values at the minimum and interpret the quantized fluctuations as particles. It might seem that only the state with the lowest energy can serve as what we perceive as the vacuum, as higher energy states would eventually decay into the lowest energy one. However, the lifetime of such a metastable state can be extremely long and certainly on the scale of the age of the universe.

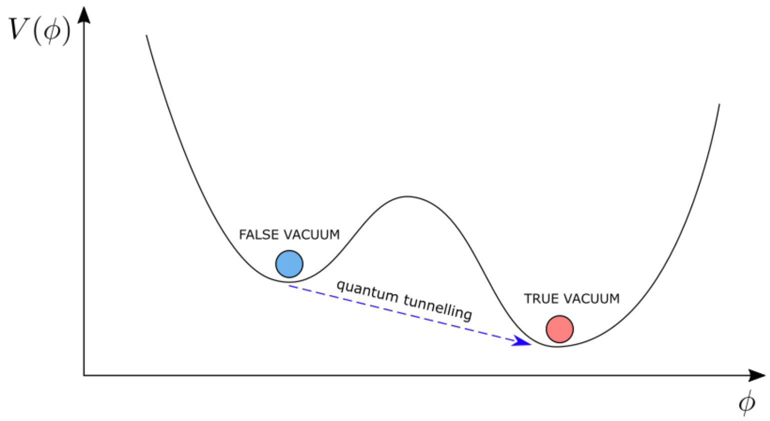

Figure 1. Potential with a local minimum (false vacuum) and a global, energetically favourable minimum (true vacuum). A phase transition from the higher energy state to the energetically lower one is possible by quantum tunnelling.

It is important to note that the present vacuum state is not determined solely by the absolute minimization of the potential of the Standard model but is dependent on the dynamical evolution of the universe from the hot and dense phase that shortly followed the Big Bang. It is therefore conceivable that during its evolution, the universe would end up in a metastable vacuum. In such a case, a bubble of true vacuum could appear somewhere in the universe and expand rapidly at the speed of light. When talking about the metastable vacuum in the Standard model, we are talking about the minimum of the potential of the Standard model at the current value, which we call the electroweak vacuum. However, it can be shown that the electroweak minimum is not the absolute minimum of the potential, if the top quark is sufficiently heavy [3]. Current experiments strongly suggest that the vacuum of the Standard model is a false vacuum, meaning that there exists a vacuum state with a lower energy, however stable vacuum is still allowed by experimental uncertainties [3] as shown in figure 2. The physics of this phenomenon is by no means settled, as new physics beyond the Standard model could still show stability. If our vacuum turns out to be

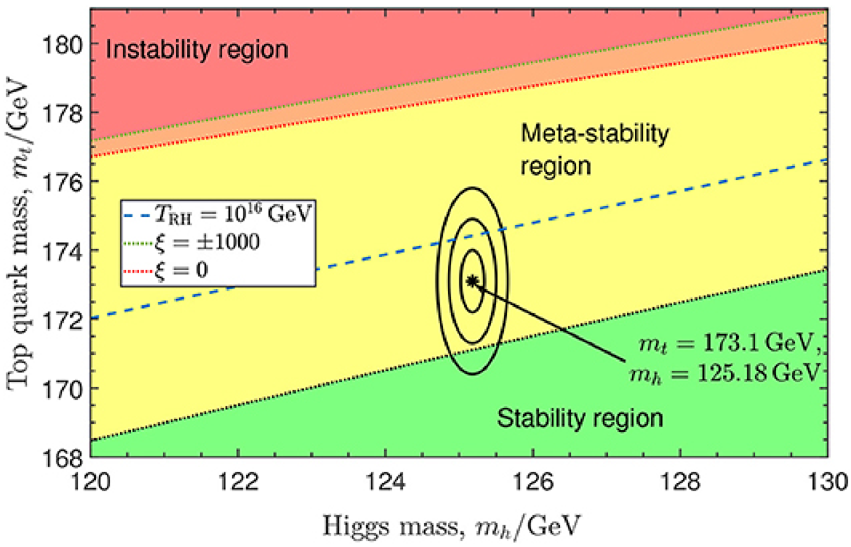

Figure 2. Stability diagram of the Standard model in terms of Higgs boson mass and top quark mass. Ellipses show the 1-3σ confidence intervals. The green region represents conditions where the vacuum is absolutely stable, the yellow region represents the metastable vacuum, meaning that the vacuum would not have decayed by the present time, but would decay in the future. Finally, the red region represents a vacuum so unstable that it would not have survived until the present day. Reproduced from [3].

metastable, the calculated lifetime of it would still be significantly longer than the current age of the universe. The current range of values for the lifetime of the universe (in the Standard model) to 95% confidence is $1 \times 10^{65}\,\mathrm{s} < \tau < 1 \times 10^{1383}\,\mathrm{s}$ [4]. This means we are 95% confident that the lifetime of our universe is at least $1 \times 10^{65}\,\mathrm{s}$ or $\sim 10^{47}$ times longer than the current age of the universe, which is ~ 13.7 billion years.

3. False vacuum decay in condensed matter physics

As the timescales of false vacuum decay in nature are astronomic and the probabilities extremely small it has proven challenging to study this phenomenon outside the realms of theory. However, even theoretical studies are limited only to certain approximations. The analogues between bubble nucleation in vacuum and the first order phase transition in condensed matter physics have opened up opportunities to study such phenomena in condensed matter systems. Platforms, such as the Ising model solved by a quantum annealer [5], IBM's digital quantum computer [6] and Rydberg atom arrays [7], could soon provide analogues for arbitrary quantum fields. Studying the dynamics and quantum effects of these systems could then be used to make quantum measurements of these systems instead of performing theoretical calculations for field theories [5]. In the context of false vacuum decay, condensed matter systems offer the distinct advantage of accessible timescales, on which measurements can be performed.

Metastable states appear in various condensed matter systems. These states can exhibit decay into lower, more energetically favourable states. One example of such systems is the current-biased Josephson junction, which exhibits macroscopic quantum tunneling out of a zero-voltage state. Such a system can be represented with a particle in a one-dimensional tilted cosine potential, with the zero-voltage state corresponding to confinement of the particle into one of the wells of the potential. The tilt of the potential is introduced by the bias current. When the particles tunnels out of this state it runs freely down the tilted potential representing a current appearing. Measurements of the escape rate at low temperatures show timescales of $\sim 10\,\mu\mathrm{s}$ [8]. Another example is the metastable phase in the electronic crystal of 1T-TaS_2. Using laser pulse excitations, the ground state ordering of the crystal is destroyed, after which the systems is trapped in a metastable state. This state decays on timescales between $1 \times 10^{-9}\,\mathrm{s}$ and $1 \times 10^{-3}\,\mathrm{s}$ to the ground state [9]. Another way of examining these phenomena is by using strongly-interacting ultracold atoms in an optical lattice. By shaking the lattice at different frequencies it is possible to create a two state system: a Mott insulator phase and a superfluid state. At lower shaking frequencies the system is in the ground state of the Mott insulator phase. When the frequency is linearly increased the ground state changes to the superfluid state, however the system gets stuck in the now metastable Mott insulator phase. The dynamics of such phase transitions were shown to occur at rates of $\Gamma \sim 1 \times 10^{2}\,\mathrm{s}$ [10]. These timescales are much more attainable to experimental measurements and simulations compared to the timescale on which vacuum decay is predicted to occur in our universe. Studying a system which would exhibit dynamics analogue to false vacuum decay in the universe but on a much shorter timescale could be used as a tool for studying the dynamics of our vacuum.

Currently, the decay of a relativistic false vacuum and observation of bubble nucleation of a true vacuum has not been achieved in a laboratory experiment. Obstacles to such realizations are the need for a system with a metastable potential and the requirements for the dynamics to be driven by quantum fluctuations, rather than thermal noise, thus guaranteeing a quantum transition not a thermal one. The second requirement is needed as relativistic vacuum decay is caused by quantum fluctuations rather than thermal fluctuations. While an experiment showing false vacuum decay in particle physics would be preferred in principle, the energies required are much higher than those that are currently accessible in particle accelerators [11].

4. Quantum Ising spin chain

Lastly, we will discuss false vacuum decay in a system described by a one-dimensional transverse Ising model with an additional longitudinal magnetic field. Using simulations of real-time dynamics following a quantum quench we can provide an environment to test the false vacuum decay scenario. Quantum quenches here refer to a sudden change of a parameter in a system's Hamiltonian.

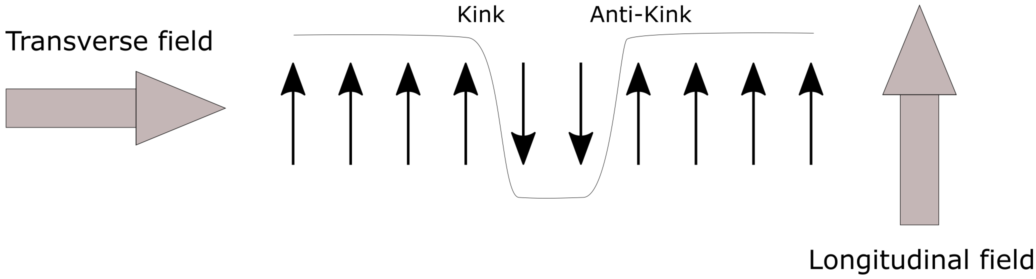

Figure 3. The illustration of the transverse Ising model in an external longitudinal magnetic field. Kinks and anti-kinks between the two domains are also shown.

We can begin by considering the equation for a quantum Ising chain in a transverse and longitudinal external magnetic fields, for which the Hamiltonian is

$$H(h_x, h_y) = -J \sum_i \left(\sigma_i^z \sigma_{i+1}^z + h_x \sigma_i^x + h_z \sigma_i^z\right),$$

where σ_i^α are the Pauli operators, h_x the amplitude of the transverse field and h_z the amplitude of the longitudinal magnetic field. The model is schematically shown in figure 3. We set $J = 1$, which represents the choice of units of energy ([J]) and time ([$\hbar$/J]). By setting $\hbar = 1$ we can work in units of time [1/J]. For a strictly transverse field Hamilitonian where $h_z = 0$, the model possesses a $\mathbb{Z}_2$ symmetry, which means that the Hamiltonian is invariant under the $\sigma^z \to -\sigma^z$ transformation. However, the ground state is not invariant under the same rotation. This means the symmetry is spontaneously broken for $|h_x| < 1$, which represents the ferromagnetic phase. At the critical value $|h_x| = 1$ the model undergoes a phase transition to a disordered state. In the ferromagnetic (ordered) phase we have two possible ground states. One with all spins along positive z with magnetization $\langle\sigma_i^z\rangle = \left(1 - h_x^2\right)^{1/8} = M$ [12] and one with all spins along negative z with corresponding magnetization $\langle\sigma_i^z\rangle = -M$. This model is exactly diagonalizable using the Jordan-Wigner transformation to map the problem to free fermions. The resulting Hamiltonian is further simplified by using the Fourier transformation to map the problem to the momentum space. Using these transformations we can see that excitations in the ferromagnetic phase are kinks between the domains with opposite magnetizations. These kinks can propagate freely and have the dispersion relation

$$\omega(\theta) = 2\left(1 - 2h_x \cos\theta + h_x^2\right)^{1/2},$$

where θ is their quasimomentum and h_x the amplitude of the transverse magnetic field. The values of θ are obtained in the diagonalization process and are $\theta = -\pi, -\pi + (2\pi)/N, -\pi + (4\pi)/N..., \pi - (2\pi)/N$. By expanding the dispersion relation for small θ we get the expression

$$\omega(\theta) = 2\sqrt{(h_x - 1)^2} + \frac{h_x \theta^2}{\sqrt{(h_x - 1)^2}} + O(\theta^4).$$

Comparing this expression to the relation for free particles $\omega = \frac{k^2}{2m}$, we can see that h_x represents the mass of the excitations in our model. In the case where the longitudinal field is non-zero

($h_z \neq 0$) the $\mathbb{Z}_2$ symmetry is broken explicitly and the two degenerate ground states are split by an energy gap $\sim 2h_z MN$, where N is the number of spins in the chain and M is the magnetization $M = \left(1 - h_x^2\right)^{1/8}$. In this case, we can talk about states that represent the true and false vacua. The state with the magnetization aligned with the external longitudinal magnetic field is the ground state of this model and represents the true vacuum. The state with the opposite magnetization with respect to the external magnetic field is a metastable state and represents the false vacuum. The additional longitudinal field also introduces a linear potential between kinks that confines them. A bubble of false vacuum of length l has an energy cost of $2lh_zM$. This energy cost is then responsible for the confinig force. For example, a pair of a kink and an anti-kink that start off in opposite directions (as is shown in SI of [13]) are pulled back by the confining force, which causes oscillations as shown in figure 4. The analogy to the binding of quarks by the strong interaction causes these bound states to be called mesons [13].

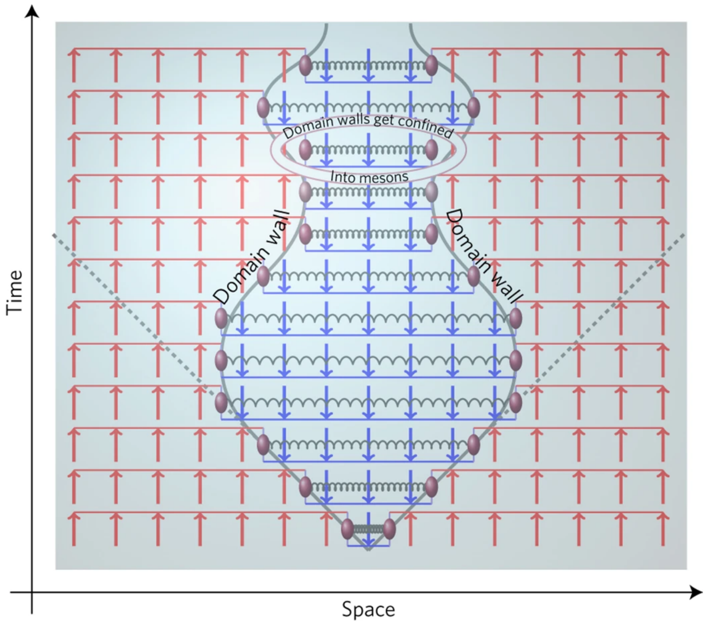

Figure 4. A bubble of false vacuum (shown in blue) is confined leading to oscillatory behaviour. Reproduced from [13].

Vacuum decay happens when the system is in the state of the false vacuum and then transitions to the true vacuum. The basic mechanism for this decay is the formation of bubbles of true vacuum. The energy needed for creating a bubble of size l is

$$E_l = 2m - 2h_zMl = \frac{f(\pi)}{\pi} - 2h_zMl, \tag{1}$$

where $2m = \frac{f(\pi)}{\pi}$ represents the masses of the two kinks, which is dependant on field h_x and $f(\theta) = \int_0^\theta \omega(\alpha)\mathrm{d}\alpha$ [14]. A bubble of false vacuum of size $\tilde{l}$ is called resonantly excited when the two terms in (1) cancel out and the energy $E_{\tilde{l}} = 0$, as seen on figure 5. A theoretical study [14] of the metastable vacuum decay in the Ising chain found the following expression for the decay rate per chain site

$$\gamma = \frac{\pi}{9} h_z M \exp\left(-\frac{q}{h_z}\right), \tag{2}$$

where $q = |f(-i \ln h_x)|/M$ and $f(\theta) = \int_0^\theta \omega(\alpha)\mathrm{d}\alpha$. It should be noted that q and M depend solely on the field h_x. The decay rate is obtained by treating the transformed free-fermion Hamiltonian perturabatively for h_z. The results are then simplified using the Fourier transform, giving us the

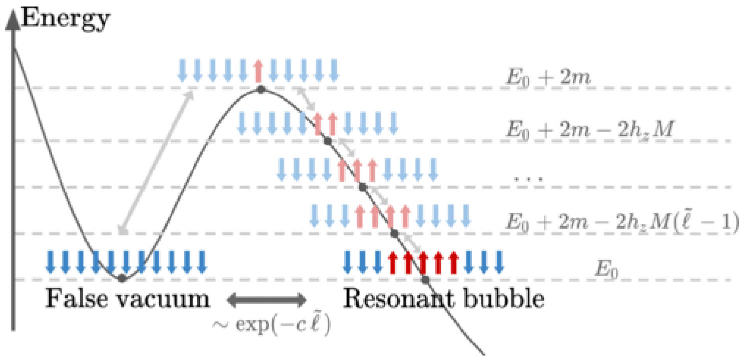

Figure 5. Bubble formation in false vacuum. False vacuum is shown in blue and true vacuum in red. Reproduced from [15].

expression for the decay rate, which transforms into the Fermi golden rule that in the limit $h_z \to 0$ gives us the expression for γ. The decay rate γ can be understood as a number of created resonant bubbles per unit time divided by the number of sites.

Such a model has been studied numerically using numerical simulations of the non-equilibrium dynamics after a quantum quench. This can be an elusive phenomena to study as there can be other effects that override vacuum decay, such as Bloch oscillations [16]. Another difficulty is finding the balance between a reasonable timescale (where (2) holds) and numerical accessibility of these timescales. With these simulations it is possible to find an interval of parameters (h_x, h_z) where false vacuum decay of the $H(h_x, h_y)$ Hamiltonian can actually be observed in simulations.
It is important to first consider the time scales of the dynamics. The first thing to happen is the creation of off-resonant bubbles in the false vacuum. During this time, which we denote as transitional time, the system still remains effectively in the false vacuum state, until resonant bubbles are created. Here we are mostly interested in the growth of resonant bubbles, as this is the process behind the false vacuum decay rate described by (2). Because of the limitations of the numerical method used, the dynamics are limited to timescales of $\sim 10/\mathrm{J}$.

To analyze the results of the numerical simulations the following two observables were analyzed

$$F(t) = \frac{\langle \sigma_i^z(t) \rangle + \langle \sigma_i^z(0) \rangle}{2\langle \sigma_i^z(0) \rangle}, \tag{3}$$

$$G(t) = 1 - ||\rho(t) - \rho(0)||, \tag{4}$$

where $\rho(t)$ is the two-site (reduced) density matrix at time t and $||\rho(t) - \rho(0)||$ the trace distance between the two density matrices. These observables were chosen to check whether both of their dynamics follow the theoretical prediction (2). At time $t = 0$ both quantities satisfy $F(0) = G(0) = 1$, and they both tend to zero in the true vacuum. The magnetization of the system is the only needed information to determine the time evolution of $F(t)$. The expected decay rate for F is therefore

$$\gamma_F \simeq \gamma \tilde{l} = \frac{f(\pi)}{18} \exp(-\frac{q}{h_z}). \tag{5}$$

The size of the resonant bubble $\tilde{l}$ is obtained by setting $E_l = 0$ in (1) and is $\tilde{l} = \frac{f(\pi)}{2h_z M\pi}$. For small values of h_z the rate γ_F is much larger than γ, which allows for smaller timescales on which the simulation needs to be performed. The rates of decay for observables F and G are obtained by fitting an exponential function

$$O(t) = A_O e^{-\gamma_O t}\,; \qquad O = F,\, G, \tag{6}$$

on the numerical values of F and G from the simulation for the appropriate time interval. This is illustrated on figure 6, where exponential decay, after a short transition period, is evident. Equation

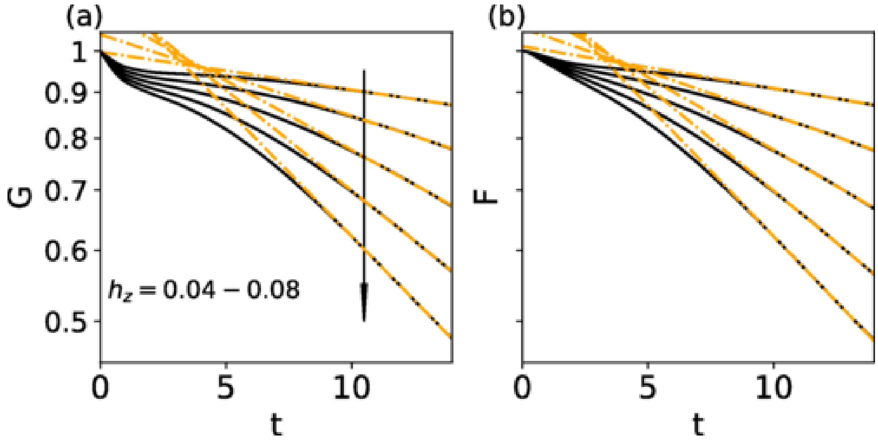

Figure 6. Time evolution of operators $F(t)$ and $G(t)$ as defined in (3) in (4) on a semi-log scale. The quench $-h_z \longrightarrow h_z$ was done at $h_x = 0.8$ and different values of h_z. The dot-dashed lines shown are the exponential fits of the decay. After a short transitionary period we can see exponential decay in all cases (exponential decay shows as a linear function on a semi-log scale). Reproduced from [15].

for the decay rate γ_F (5) predicts exponential dependence on $1/h_z$, which can be clearly seen by an exponential fit of obtained decay rates

$$\gamma_O = k_O \exp\left(-q_0/h_z\right) \; ; \qquad O = F,\, G, \tag{7}$$

as can be seen in figure 7. In the figure 7-d a comparison of obtained coefficients q_F and q_G and the theoretical prediction $q = |f(-i \ln h_x)|/M$ is presented. Evidently, the numerically obtained coefficients agree with the prediction. In the panels 7-a,b,c the decay rates of γ_F, γ_G and the theoretical prediction (5) are plotted against $1/h_z$ at different values of h_x. While the q coefficients of F and G match predicted ones, the prefactors k_F and k_G differ from the prediction, which is obvious in the vertical shift. This shift is not surprising as the prefactors depend on the specific observable measured. An example is the comparison of (2) and (5). In addition, these prefactors are affected by the approximations done to obtain (5).

These numerical findings show that a range of parameters (h_x, h_y) exists, such that the decay rate of false vacuum is accessible in measurable timescales. Numerical protocols used could serve as the basis for experimental examinations. One possibility is trapped-ion experiments which have already shown particle confinement in spin chains [17]. Another avenue of further research is simulations of this phenomenon using quantum processes such as quantum annealing. Devices such as D-Wave quantum annealer can directly simulate the transverse Ising model with an additional longitudinal external field. These simulations can probe the systems on far longer timescales of ~ 1000/J, which would open up the window of parameters where false decay rate occurs. Numerical simulations are limited as h_x decreases γ increases, as described by (2), and so the timescale needed for the decay greatly lengthens. Simulations on quantum annealing platforms could therefore check theoretical predictions for smaller values of h_x.

5. Conclusion

Since the latest measurements of the masses of elementary particles suggest that our vacuum lies in the metastable state, the interest in a more detailed understanding of the phenomena has increased. Quantum simulators have opened up a new way of studying problems in quantum field theory, including metastable vacuum decay. One such system is the transverse Ising model. Numerical simulations have shown that it can exhibit vacuum decay for certain parameters following a quantum quench. These findings show the potential for studying quantum chains and their non-equilibrium

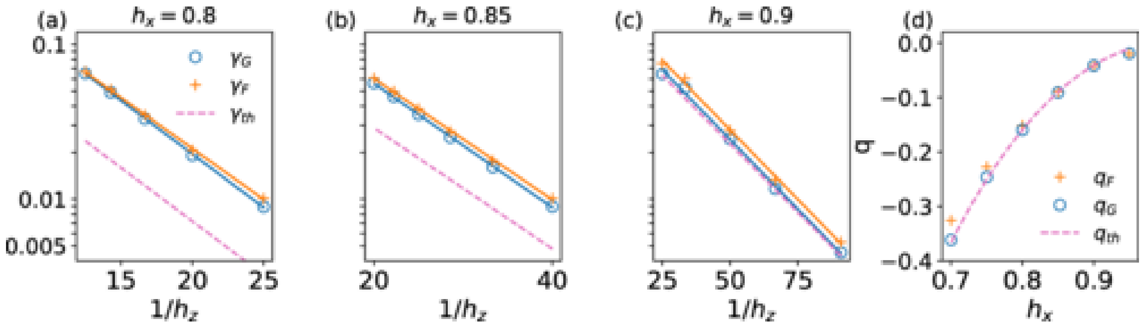

Figure 7. In (a), (b) and (c) the fitted decay rates γ_F and γ_G are shown. These are shown in continuous lines and are obtained from fits shown in figure 6. The dashed line is the theoretical prediction from (5). In panel (d) the values of coefficients q_F and q_G obtained from fits are compared with the theoretical value of $q_{th} = |f(-i \ln h_x)|/M$. Reproduced from [15].

dynamics. In particular, these simulations provide a way to directly observe non-equilibrium dynamics and the decay rate of the false vacuum. This is especially important as studying these effects analytically has proven quite difficult in cosmology.

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