)

CONNECTION OF ELONGATION OF GALAXY CLUSTERS WITH OTHER PROPERTIES

EMA MLINAR

Fakulteta za matematiko in fiziko
Univerza v Ljubljani

Galaxy clusters, the largest known virialized objects in the universe, are known to be triaxial. In this paper the triaxiality of simulated galaxy clusters and its correlation with elongation, mass, dynamical state and evolution with time are presented.

Zoomed-in simulations of five different regions of a larger box were used. The projected simulated clusters are in good agreement with observations. A relation between average 2D ratio of semi-axes and maximal elongation of 3D clusters was then established, and using this a theoretical limiting value for cluster maximal elongation was found.

In the second part the relation between rate of interaction of the cluster (its dynamical state) and its elongation was studied, confirming the findings by other authors. Since more massive clusters are frequently in interaction, they are on average more elongated. There is a clear trend of clusters getting on average more elongated with time. A weak correlation between time evolution of elongation and cluster final mass was found, more massive clusters are on average getting more elongated with time, while smaller clusters are getting more spherical.

POVEZAVA MED PODOLGOVATOSTJO JAT GALAKSIJ IN OSTALIMI LASTNOSTMI

Jate galaksij so največje znane strukture v vesolju, za katere velja virialni teorem. Njihova pomembna značilnost je oblika, znano je namreč, da imajo obliko elipsoida (ki ima vse tri osi različno dolge). V tem članku je predstavljena povezava med obliko in maso, stopnjo dinamike in obnašanjem v času.

S pomočjo zoom-in simulacij petih različnih podobmočij je pokazano, da se podatki iz simulacij zelo dobro ujemajo z opazovalnimi podatki. Mogoče je določiti povezavo med povprečnim 2D razmerjem polosi ter maksimalno podolgovatostjo v 3D. S pomočjo znanih podatkov se tako da določiti omejitev za podolgovatost jate v 3D, ki se zelo dobro ujema s podatki iz simulacij.

Rezultati potrjujejo izsledke drugih avtorjev, da so jate v trku bolj podolgovate. Prav tako so tudi masivnejše jate v povprečju bolj podolgovate. V povprečju jate postajajo s časom bolj podolgovate, s tem da tu igra pomembno vlogo masa jate. Masivnejše jate namreč postajajo v povprečju bolj podolgovate s časom, medtem ko manj masivne jate postajajo bolj sferične.

1. Introduction

Galaxy clusters are the largest known virialized objects in the universe. Therefore they are very useful probes to study cosmological parameters. We can use the data to determine whether the universe formed hierarchically or not. Modern observations support the hierarchic model of structure formation but there is still much work to be done in this field. Furthermore, there is not much known about dark matter and dark energy, and galaxy clusters are one of the rare probes that allow us to study such open questions in cosmology.

Galaxy clusters are largely dominated by dark matter, while around 10% of their mass is in the form of an ionized plasma (ICM - intracluster medium) and a few percent in the form of stars and galaxies. Galaxies and gas in clusters are normally studied using Sunyaev-Zeldovich effect or gravitational lensing or X-rays coming from central parts of galaxy cluster. Usually all these data is combined to get a better picture of a single galaxy cluster.

Despite many authors still assuming sphericity for simplicity, galaxy clusters are known to be triaxial objects (see [1] for a review). Throughout history there were different attempts to study the shape of astronomical objects. One of the first was related to galaxies. Hubble in 1926 tried to determine the frequency of galaxies with given ellipticity, assuming they are randomly oriented in the sky. Many authors also assume that clusters are either prolate or oblate. Prolate clusters are elongated at the poles, as opposed to oblate ones that are flattened at the poles. [2] for example reports preferred prolate shapes in a sample of 25 observed clusters. Statistical methods were used for different astronomical objects, with the result that prolate-like objects are more frequently found on larger scales.

In this short paper we present the comparison between our projections of spheroids and real observational data. We also establish a relation between average 2D elongation of a projected ellipse and the maximal 3D elongation. Then we investigate how cluster elongation is connected with cluster's mass, how clusters evolve with time and how interaction between galaxy clusters influences their shape. This is the first time this specific simulations (described below) are used to determine the shape of simulated galaxy clusters.

Why the shape of galaxy clusters is important can be seen in this example. Precision astronomy and precision cosmology rely on the accurate determination of galaxy cluster masses. Masses are important when we want to trace the formation of cosmic structure and its evolution. However, galaxy cluster masses are not a directly observable quantity. Measured quantities (such as surface brightness, gas density, gas temperature etc.) are extracted from 2D images of a galaxy cluster and under some reasonable assumptions (e.g. sphericity, hydrostatic equilibrium) turned into full 3D information. The underlying assumptions strongly depend on the cluster's intrinsic shape and its orientation. As reported by [2] (see also references therein), if clusters are elongated along the line of sight their computed masses will be higher with respect to their true masses. A simultaneous analysis of galaxy cluster data in different wavelengths (see [2]) is useful to obtain a coherent 3D result. This kind of detailed observations require however a large amount of observing time on very different facilities (X-ray telescopes, radio telescopes etc.) and are not feasible for a large amount of clusters, especially for the distant ones, that are not well resolved in images.

2. Simulations

We used simulations described in [3]. The complete set of simulations used in previous papers is composed of 29 hydrodynamic zoom-in regions that evolve in a ΛCDM cosmology with parameters: $\Omega_m = 0.24$, $\Omega_b = 0.037$, $n_s = 0.96$, $\sigma_8 = 0.8$ and $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Ω_m is the matter density parameter, Ω_b barion density parameter, n_s primordial spectral index, σ_8 is the amplitude of the power spectrum of the density fluctuations, and H_0 is the Hubble constant. At first the regions are extracted from a larger dark matter only simulation, then the regions are zoomed-in and re-simulated using baryons and at a greater resolution. The simulations are carried out with the code GADGET-3 (see [4]). For the hydrodynamic description an updated formulation of SPH (Smoothed-particle hydrodynamics) is used including higher-order interpolation kernels and derivative operators, and advanced formulations for artificial viscosity and thermal diffusion. For the analysis we are presenting in this paper, we limited our sample of regions to five regions (D1, D5, D6, D9 and D22). All the regions were simulated including gas cooling, formation of new stars and active galactic nuclei (AGN) feedback from supermassive black holes (SMBH). The AGN feedback is modelled as thermal feedback that includes mechanical outflows and radiation. The transition between the radio and quasar feedback mode is accomplished by modelling gas accretion rate and SMBH mass appropriately (see [5]).

In each of the five selected regions we obtained 30 clusters with masses above $3 \cdot 10^{12} M_\odot/h$, where h is dimensionless Hubble constant defined with an expression $H_0 = 100\,\mathrm{h\,km\,s^1\,Mpc^1}$.

2.1 SUBFIND

We used an algorithm called SUBFIND to determine the locations and sizes of galaxy clusters. In the beginning the algorithm extracts overdense, self-bound particle groups within a larger parent group. At first a local estimate of the density at the positions of all particles is made. All overdense regions are considered to be substructure candidates. After defining substructures based on spatial distribution of particles, the algorithm eliminates particles with positive energy, so that only bound particles remain. If more than a minimum number of particles remain we reclaim it as subhalo. Subhalo centre is defined as the position of the most bound particle. Note that one particle can be a member of several groups. For more details see [6].

3. Methodology

The aim of our research was to determine the shape of galaxy clusters and their projections onto a 2D plane in order to compare them with simulated data. We used simulated data for dark matter to determine the properties of halos of galaxy clusters. We assumed that galaxy clusters are ellipsoids, which is a good approximation for dark matter halos, but not necessarily for the distribution of gas and galaxies.

In order to get data for a single cluster we extracted the data from a sphere of a virial radius centered on the clusters found previously in SUBFIND in our simulation. Then we computed the semi-axes of the cluster ellipsoid.

3.1 Determining the semiaxes of ellipsoid

As in [7], we determined the semi-axes as follows. We arrange a matrix:

$$M = \sum_i \begin{bmatrix} x_i^2 & x_i y_i & x_i z_i \\ x_i y_i & y_i^2 & y_i z_i \\ x_i z_i & y_i z_i & z_i^2 \end{bmatrix}$$

where we count values for every particle. Eigenvalues of this matrix are proportional to the square of the semi-axies of the ellipsoid.

3.2 Projection of 3D ellipsoid onto a plane

We have an ellipsoid with semi-axes $a : b : c$ and a plane through coordinate origin with a normal vector $\vec{n} = (n_1, n_2, n_3)$. Since we are interested only in the average ratio of semi-axes, we can, without loss of generality, assume that our ellipsoid is aligned with the coordinate system (i.e. eigenvectors are parallel to coordinate axes). So it satisfies the equation:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1,$$

We will now project the ellipsoid onto a plane. It is known that projection onto every plane can be written as:

$$\vec{v} - (\vec{v} \cdot \vec{n})\, \vec{n} = Proj\,(\vec{v}) = (X, Y, Z)$$

where $\vec{v}$ is a vector that we project and $\vec{n}$ the normal unit vector of the plane. Since $\vec{v}$ lies on an ellipsoid, we can write it as follows:

$$\vec{v} = \left(x, y, \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2} \right).$$

Note that the third coordinate of the vector could also have a negative sign, but for our analysis, we will now consider only the plus sign. Later we will just reflect the solution over the origin of the coordinate system. Thus we got an ellipse in 3D space. We will multiply the vector by the matrix:

$$A^{-1} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\psi) & 0 & \sin(\psi) \\ 0 & 1 & 0 \\ -\sin(\psi) & 0 & \cos(\psi) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\psi)\cos(\theta) & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} =$$

$$\begin{bmatrix} \frac{n_3 n_1^2 + n_2^2}{n_1^2 + n2^2} & \frac{n_1 n_2 (n_3 - 1)}{n_1^2 + n2^2} & -n_1 \\ \frac{n_1 n_2 (n_3 - 1)}{n_1^2 + n2^2} & \frac{n_3 n_2^2 + n_1^2}{n_1^2 + n2^2} & -n_2 \\ n_1 & n_2 & n_3 \end{bmatrix}.$$

Thus we rotate it to an ellipse in x-y plane, where we got angles ϕ, θ and ψ using evaluation. For every point we calculate its distance r to coordinate origin. It reads:

$$r = |\vec{r}| = \sqrt{X^2 + Y^2},$$

so we just forget the Z component. We can easily find the most distant point (from the coordinate origin) using any of the maximization methods. Using coordinates of the most distant point we can determine the angle φ (see Figure 1), and notice that semi-minor axis is just at angle $90° + \varphi$.

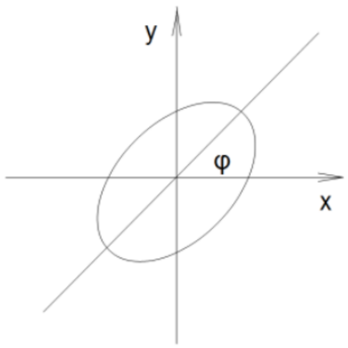

Figure 1. Ellipse, centered on the origin of x and y axes and rotated by an angle φ.

With this method we calculate the ratio of semi-axes for each of the selected cluster in our simulated regions:

$$R = \frac{r(\varphi)}{r(\varphi + 90°)}.$$

4. Results

4.1 Distribution of clusters according to their elongation

The distribution of clusters according to their elongation is shown in Figure 2. We denoted axes by a, b, c from the longest to the shortest axis. As an indicator of elongation, we took ratios b/a and c/a. Note that b is always greater that c (this is why all of the clusters are located in the bottom part, below the line of equality).

We can see that the majority of clusters are not perfectly spherical. They instead have a maximum distribution at semi-axes ratios of around 0.9. We also notice that there are some clusters with an extreme ratio of axes (i.e. 0.2). Further analysis shows these are in most cases massive

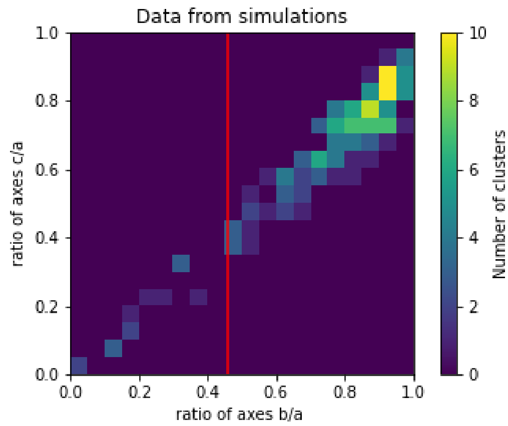

Figure 2. Distribution of elongation in simulated galaxy clusters. As an indicator of elongation we use the ratios between the semiaxes b/a and c/a. Color represents the fraction of clusters with certain ratio, extending from yellow (maximal) to dark blue (minimal). The meaning of a red line will be explained later (see chapter 4.3), it represents the maximal elongation determined using Figure 5.

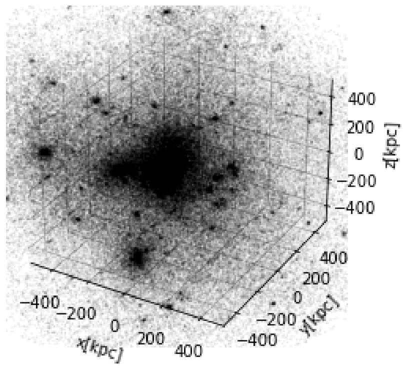

Figure 3. An example of an elongated galaxy cluster that is in a merger. These clusters are usually massive clusters in the middle of zoomed-in areas.

clusters (see Figure 6). Such galaxy clusters grow by merging with other clusters and have a high interaction rate, therefore, in most cases, their elongation is the result of a recent merger. There is an example of such a cluster in Figure 3.

4.2 Comparing projected clusters to observational data

Simulated galaxy clusters were projected onto a plane using the method described in section 3.2 Then we compared the projected data with observations obtained from [8]. Authors of the paper used the data of 25 different galaxy clusters to compare the X-ray, lensing and SZE (Sunyaev-Zeldovich effect) morphological properties. Since the data using these three methods are in good agreement with each other, they represent a robust observational data. The comparison between simulated and observational data is shown in Figure 4. We see that there is a good agreement between the two. However, there are no observed clusters with axes ratio less than 0.6. The reason for this may lie in the different sample (observations were conducted only on high mass clusters), the observable part (observations include only the innermost radius, not the virial one) and on how

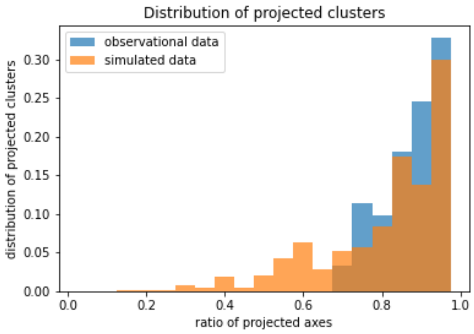

Figure 4. Comparison of observational data with simulated projected galaxy clusters. Observational data from [8] are reported for 25 massive clusters, while in simulations we selected 30 clusters with masses above $3 \cdot 10^{12} M_{\odot}/h$ and projected them onto every plane.

the clusters look in observations with respect to simulations. In observations we see two separated cores when looking in i.e. X-ray, while they have the same outer part when observing dark matter distribution in simulations, since it is known that dark matter halos are much larger than the observable inner part of the clusters.

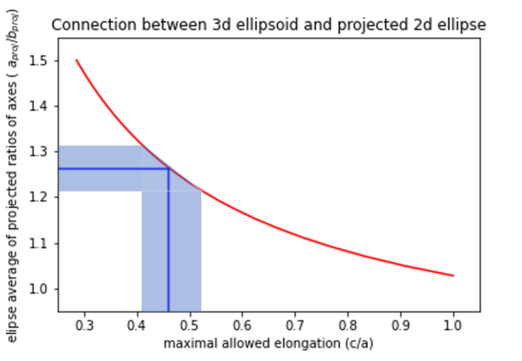

Figure 5. Numerical relation between maximal elongation of 3D ellipsoid and average ratio of projected axes in 2D. Blue line represents the value of average projected ratio estimated from projected simulated data - 1.26 ± 0.05, and the corresponding value of maximal allowed elongation in 3D - 0.46 ± 0.06.

4.3 Correlation between the maximal ratio of axes of 3D ellipsoid and the average ratio of semi-axes of 2D ellipse

We studied the correlation between the maximal ratio of semi-major and semi-minor axes of a 3D ellipsoid and the average ratio of semi-axes for a 2D projected ellipse. We projected 3D ellipsoids onto a plane as described in section 3.2 We averaged over all planes and all ratios of semi-axes up to a certain maximal ratio of semi-major (a) and semi-minor (c) axes. Thus we got a numerical relation between above variables (see Figure 5). In other words, we assumed Heaviside distribution, so that there is uniform cluster probability density up to a certain value and probability density 0 everywhere else. We got the average value of ratio of axes of 1.26 ± 0.05 from simulated data. Then from this graph it follows that a value of 0.46 ± 0.06 is the limit for maximal elongation in 3D.

We tested it for the average ratio of semi-axes of 2D ellipse obtained from projected data from simulations. We found that there is a surprisingly good agreement with simulated data for maximal ratio of axes of 3D ellipsoid (see the red line in Figure 2). There seems to exist a sharp limit under which clusters are located. This limit represents the maximal allowed elongation.

However, there are clusters below this limit. We can argue that these galaxy clusters are not in equilibrium, since more elongated clusters are more likely to be in interaction (see chapter 4.5).

4.4 Correlation between mass and elongation

We found a weak correlation between mass and elongation (see Figure 6). We can see that more massive clusters are on average more elongated. As elongation we took the ratio c/a, where a, b, c are semi-axes with decreasing values. For cluster mass we used the virial mass.

This correlation can be explained by massive clusters are more likely being in merger with other clusters and thus looking more elongated during or after a merger (see section 4.5).

Our findings about the connection between mass and elongation are consistent with [9]. The authors researched investigated the influence of the dynamical state and the formation history on both the morphology and local connectivity of about 2400 groups and clusters of galaxies from the large hydrodynamical simulation IllustrisTNG at z=0, where they found that massive halos are significantly more elliptical and more connected to the cosmic web than low-mass ones.

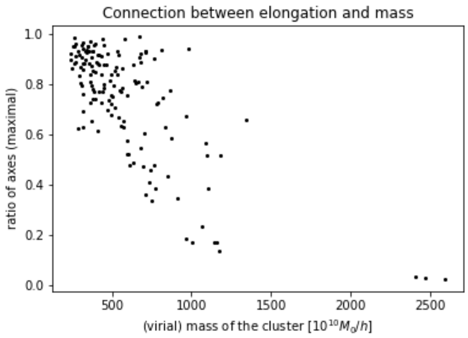

Figure 6. Connection between mass and elongation. From above graph we can conclude that the more massive clusters are more elongated than the less massive ones.

4.5 Correlation between interaction and elongation

We also tried to determine whether there exists a relation between elongation of galaxy clusters and their dynamical state (whether they have recently merged or interacted). As an indicator of interaction between galaxy clusters we used the separation between the center of the cluster (defined as the minimum of the potential) and the center of the most massive galaxy in it (also called BCG - brightest cluster galaxy). We expect disturbed clusters to have their most massive central galaxy displaced from the center of the potential well. Similarly to other works (e.g. [10]) the dynamical state of a cluster (defined as regular or disturbed) is defined by using the center shift. The center shift is defined as the spatial separation between the position of the minimum of the potential and the center of mass of the galaxy cluster. Elongation is measured as the ratio between greatest and lowest semi-axis of the galaxy cluster.

In Figure 7 we plotted the relation between elongation and dynamical state of the clusters for data from simulations D1, D5, D6, D9 and D22. Data are obtained for redshift z=0. We can see a weak correlation between cluster elongation and dynamical state. Galaxy clusters in interaction are on average more elongated. This can be explained with mutual gravitational forces pulling each of the components apart (see [11]) or that we see two clusters that recently undergone a merger as a single elongated one.

Our findings about interaction-elongation relation is consistent with [9]. The authors studied approximately 2400 groups and clusters of galaxies from the hydrodynamical simulation IllustrisTNG

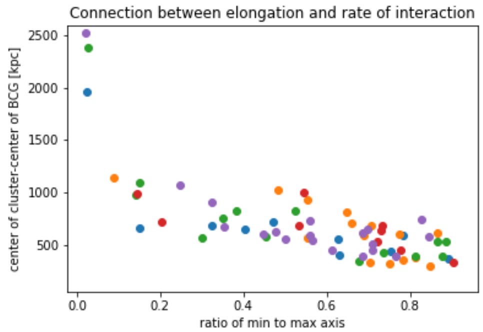

Figure 7. Correlation between the degree of interaction of a cluster and its elongation. For an indicator of interaction we took the difference between center of the cluster and center of the most massive galaxy in it. Data includes galaxy clusters in simulations D1, D5, D6, D9 and D22, each of the regions represented by a different color.

at z=0, concluding that galaxy clusters in interaction are strongly affected by the infalling materials from filaments, while relaxed clusters are not and are thus more spherical. They explained the relation between interaction and elongation with different accretion histories.

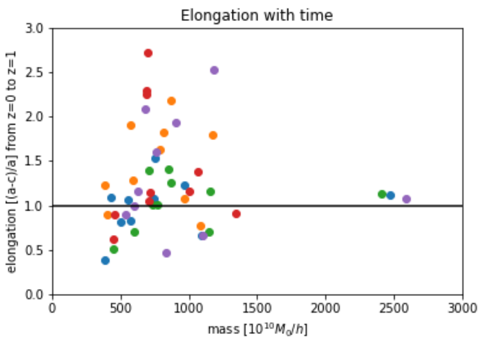

Figure 8. Time dependence of elongation of clusters with respect to their mass. As an indicator of elongation we used ratio $(a-c)/a$, where a is the greatest semi-axis and c the smallest semi-axis. Then we compare this ratio at redshift 0 and redshift 1. If the ratio reported on the y-axis is greater than one, cluster is getting more elongated with time, a lower value means that cluster is getting more spherical with time.

4.6 Time dependence of elongation of clusters

Using simulations at different redshifts we were able to determine how clusters develop with time (see Figure 8). Elongation was determined similarly as in the previous paragraph. Although some clusters are getting more elongated with time and some of them less elongated, there is a clear trend showing that on average clusters are getting more elongated with time.

There is also a trend related to cluster masses: on average the most massive clusters become more elongated with time, while smaller clusters are getting less elongated (see Figure 8).

We can also notice that three most massive clusters are evolving quite differently from others. This trend is also present in Figure 6, where we present the relation between the ratio of axes (maximal) and the virial mass of clusters. Those three clusters seem to stay as elongated as at the beginning, but they have a high interaction rate (as seen in Figure 7).

5. Conclusion

In this paper we studied the triaxiality of simulated galaxy clusters and tried to connect their shape to other properties (namely masses and dynamical state). We have shown that data from simulations fits well with observational data regarding the ratio of projected axes. We established a relation between average ratio of projected axes and maximal 3D elongation. There is still an open question about sharp limit that represents maximal (possible) elongation for (simulated) galaxy clusters. And also on the contrary, why there exist galaxy clusters below this point. One of possible explanations is that those galaxy clusters are not in equilibrium.

We confirmed that more massive clusters are normally more elongated, and also, that clusters that are in a disturbed dynamical state are more likely to have a bigger axes ratio. During the cosmic evolution most clusters seem to get more elongated. There is a weak correlation between cluster mass at z=0 and the time dependence of the elongation.

REFERENCES

[1] Limousin, M. and 6 colleagues, *The Three-Dimensional Shapes of Galaxy Clusters.* Space Science Reviews 177, 155–194 (2013).

[2] Sereno, M., De Filippis, E., Longo, G., Bautz, M. W., *Measuring the Three-dimensional Structure of Galaxy Clusters. II. Are Clusters of Galaxies Oblate or Prolate?.* The Astrophysical Journal 645, 170–178 (2006).

[3] Planelles, S. and 13 colleagues, *Pressure of the hot gas in simulations of galaxy clusters.* Monthly Notices of the Royal Astronomical Society 467, 3827–3847 (2017).

[4] *Gadget code*, Webpage: https://wwwmpa.mpa-garching.mpg.de/gadget/ [Access: 15.4.2022].

[5] Steinborn, L. K., Dolag, K., Hirschmann, M., Prieto, M. A., Remus, R.-S., *A refined sub-grid model for black hole accretion and AGN feedback in large cosmological simulations.* Monthly Notices of the Royal Astronomical Society 448, 1504–1525 (2015).

[6] Springel, V., White, S. D. M., Tormen, G., Kauffmann, G., *Populating a cluster of galaxies - I. Results at [formmu2]z=0.* Monthly Notices of the Royal Astronomical Society 328, 726–750 (2001).

[7] Gouin, C., Bonnaire, T., Aghanim, N, *Shape and connectivity of groups and clusters: Effect of the dynamical state and accretion history.* Astronomy and Astrophysics 651 (2021).

[8] Donahue, M. and 12 colleagues, *The Morphologies and Alignments of Gas, Mass, and the Central Galaxies of CLASH Clusters of Galaxies.* The Astrophysical Journal 819 (2016)

[9] Gouin, C., Bonnaire, T., Aghanim, N., *Shape and connectivity of groups and clusters: Effect of the dynamical state and accretion history.* Astronomy and Astrophysics 651 (2021).

[10] Biffi, V. and 9 colleagues, *On the Nature of Hydrostatic Equilibrium in Galaxy Clusters.* The Astrophysical Journal 827 (2016).

[11] Gnedin, O. Y., *Tidal Effects in Clusters of Galaxies.* The Astrophysical Journal 582, 141–161 (2003).

[12] Cui, W. and 8 colleagues, *How does our choice of observable influence our estimation of the centre of a galaxy cluster? Insights from cosmological simulations.* Monthly Notices of the Royal Astronomical Society 456, 2566–2575 (2016).