DARK MATTER AND POSSIBLE PARTICLE SOLUTIONS

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The hypothesis of dark matter is entertained by stating some corresponding observational evidence. Following that line of thought, using the standard cosmological principles and gravitational laws, the Friedmann equations are obtained in terms of energy densities. That is called-for, since the values of energy densities are estimated using the standard cosmological model, where dark matter is included. After a brief discussion of possible candidates for dark matter, the weakly interacting massive particles (WIMPs) are treated in more detail. They are considered as a favourable candidate because the assumption that dark matter has a relic density, after decoupling in the early stages of the Universe, is in harmony with the properties of WIMPs. Finally, a brief summary covers the ongoing experimental search for the direct and indirect detection of dark matter.

TEMNA SNOV IN MOŽNE REŠITVE Z DELCI

Hipotezo o temni snovi podpremo z navedbo nekaterih ustreznih opazovalnih dokazov. Z uporabo standardnih kozmoloških načel in gravitacijskih zakonov dobimo Friedmannove enačbe v obliki gostot energije, katerih vrednosti so trenutno ocenjene z uporabo standardnega kozmološkega modela, v katerega je vključena temna snov. Med možnimi kandidati za temno snov obravnavamo šibko interagujoče masivne delce (WIMP). Ti veljajo za ugodnega kandidata, saj je predpostavka, da ima temna snov po razpadu v zgodnjih fazah vesolja reliktno gostoto, skladna z lastnostmi WIMP-ov. Na koncu podamo kratek povzetek trenutnega eksperimentalnega iskanja za neposredno in posredno odkrivanje temne snovi.

1. Introduction

There is a general consensus today that there is more matter than what we can see in the Universe (Particle Data Group et al. 2020, Section 27; Planck Collaboration VI et al. 2018, Section 3; WMAP Collaboration et al. 2013, Section 2.1). More technically, there is a discrepancy between the mass that we measure in the Universe using electromagnetic radiation,[1] i.e. luminous mass,[2] and the mass that we indirectly measure in the Universe using standard gravitation laws,[3] i.e. dynamical mass. Assuming that there are no game-changing errors in the measurements[4] then it follows that the discrepancy can be solved by one of the following hypotheses:

1. There exists additional non-luminous mass in the Universe—hence the name dark matter (DM).
2. The laws of gravity need to be modified.[5]
3. A combination of the above.

In this paper we consider only the first hypothesis. We begin in Section 2, where we entertain the DM hypothesis by stating some observational evidence. In Section 3, we discuss the possible nature of DM, listing a few candidates. To illustrate the standard cosmological model which includes DM,

[1]Our current understanding of the Universe outside the Solar System comes almost exclusively from observations of electromagnetic radiation, with only some additional input from neutrinos and cosmic rays (Carroll 2014, p. 315).

[2]Inferred, for example, by the *mass-luminosity relation*, to give some context. See, for example, Duric 2004, Section 1.3.8.

[3]Referring to Einstein's law of gravity, or its classical limit as an approximation, Newton's law of gravity.

[4]The discrepancy lends itself to several cases of analysis, but in this paper it is taken for granted that both groups of measurements are correct in their respective relevant orders of magnitude; only their framework will be described.

[5]The most notable example of this hypothesis is the MOND paradigm first put forward by Milgrom 1983. For an account of the current gloomy status of MOND hypothesis, see the last paragraph in Particle Data Group et al. 2020, Section 27.1, and the references therein.

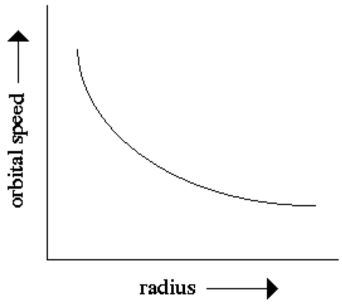

Figure 1. A cartoon picture showing how the orbital velocity $v(R) \sim 1/\sqrt{R}$ should be decreasing with increasing radius. Adapted from Schombert 2019.

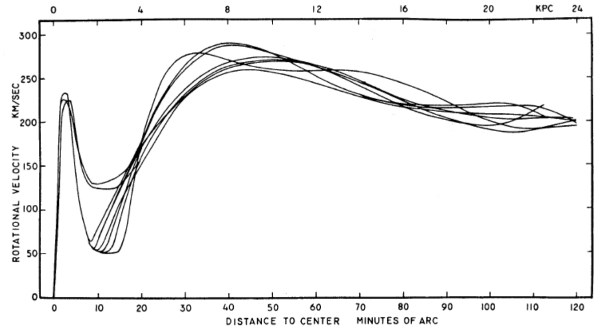

Figure 2. Fourteen rotation curves for the Andromeda galaxy (also called M31), showing the measured orbital velocities as a function of radius from the nucleus (center) of the galaxy. Contrary to what we expect far from the center, the orbital velocity is not decreasing. Taken from the pioneering paper, Rubin et al. 1970, p. 17.

we cover the dynamics of the universe in Section 4. This sets the stage for Section 5., where we discuss the thermal history of the universe which would have to explain the abundance of DM today. There, the punchline is the so-called WIMP miracle. In Section 6, we give a brief summary of the experimental progress in detecting DM. Finally, in Section 7, we finish up with a few closing remarks.

2. Dark matter (DM) hypothesis

The DM hypothesis has an elaborate history. There are many indirect cosmological observations that indicate that the luminous mass is not enough to make up for the cosmological structures that we see in the Universe. Below we will discuss three of such observational evidence in more detail. We note that even though we consequently talk about the whole Universe, the DM problem can be said to be of two categories: missing mass within a galaxy and missing mass between galaxies; either will be apparent from the context.

2.1 Galaxy rotation curves

This is a technical term for the curve describing the dependence of the velocity $v(R)$ of stars in a galaxy, as a function of the distance R from the galactic center. In other words, it describes how fast the galaxy's stars rotate around its center.

To represent the basic idea let us assume Newtonian mechanics and spherical symmetry. The centrifugal acceleration of a star, orbiting at distance R from the galactic center, must be provided by the gravitational force, i.e.

$$\frac{v(R)^2}{R} = \frac{GM(R)}{R^2}, \tag{1}$$

where G is the gravitational constant and

$$M(R) = 4\pi \int_0^R \rho(r) r^2 \, \mathrm{d}r, \tag{2}$$

means that $M(R)$ is the mass enclosed inside a sphere of radius R with a density distribution $\rho(r)$. Far from the center of the galaxy, we would expect that $M(R)$ is roughly constant—most of the luminous matter is concentrated at the center. With that assumption in mind, solving Eq. (1) for $v(R)$ we see that the velocity drops off as $v \sim 1/\sqrt{R}$, which denotes our rotation curve (see Fig. 1).

Flipping this on its head, we should see that rotation curves provide a way to measure the dynamical mass. The way to infer the mass of the galaxy is by first measuring the velocities of stars and then using Eq. (1) now to solve for the mass as a function of radius. Nonetheless, measuring such velocities is not a trivial task. The method of measuring them was pioneered by Rubin et al. 1970.[6] In technical terms, by measuring a spectrum of the galaxy at various positions along its major axis one can measure the rotational velocity at each point on the axis as a fraction of the speed of light, and that gives a resulting graph which is the galaxy's rotation curve. The graph of these observations differs from the results we expect from the distribution of luminous mass. In Fig. 2, observationally, far from the center of the galaxy we see $v(R) \approx$ const., which does not agree with our expectation. Then, from Eq. (1) we can conclude that a constant rotation curve requires $M_{\mathrm{DM}}(R) \sim R$. From Eq. (2), such a result is obtained if the density distribution of DM is

$$\rho_{\mathrm{DM}}(r) \sim \frac{1}{r^2},$$

which is known as the dark matter halo. As is understood from the context, a dark matter halo is a hypothetical region which contains gravitationally bound matter, including DM.

2.2 Galaxy clusters

A galaxy cluster is a structure that consists of hundreds or thousands of galaxies that are bound together by gravity. The average mass per galaxy in a cluster can also be determined by dynamical means. By measuring the radial velocities of the galaxy clusters members, the velocity dispersion of a cluster can be estimated and used to derive the cluster's mass from the virial theorem. Assuming that the cluster in question is a gravitationally bound and well-relaxed system, we have (Kolb et al. 1990, p. 18)

$$GM = \frac{2\left\langle v^2\right\rangle}{\left\langle r^{-1}\right\rangle},$$

where M is the cluster mass; $\left\langle v^2\right\rangle$ is the mean square velocity; and $\left\langle r^{-1}\right\rangle$ is the mean inverse separation between galaxies. For a Coma cluster, which is a group of ≈ 1000 galaxies within a radius of $\approx 11 Mpc$ (as studied by Zwicky in 1933), by using the mass-luminosity ratio the dynamical mass resulted to be 300 times greater than what was expected from the luminosity measurements (Simone 2019, p. 17). This means that most of the matter is not luminous, supporting the DM hypothesis.

2.3 Estimated density in our Universe

Let us briefly state the energy density budget of our Universe, postponing the detailed discussion for all the technical terms in Section 4. Postulating the existence of DM and dark energy, the standard cosmological model, which is called the ΛCDM model (Λ being the symbol for the cosmological constant—dark energy, and CDM being an acronym for cold dark matter, i.e. non-relativistic dark matter), concords well with the CMB measurements. Measurements from Planck Collaboration VI et al. 2018 show the following energy budget for our Universe[7]

$$\Omega_{\text{curvature}} = 0.00, \quad \Omega_{\text{matter}} = 0.32 = \begin{cases} \Omega_{\text{baryonic}} = 0.05, \\ \Omega_{\text{CDM}} = 0.27, \end{cases} \quad \Omega_\Lambda = 0.68,$$

adding up to $\Omega_{\text{tot}} = 1$. Namely, our Universe is flat (the curvature term does not contribute to the energy budget), matter contributes 32% of the budget (split into 5% of baryonic matter and 27%

[6] For an attributed article to Vera Rubin, read Childers 2019.

[7] For our purposes, the values shown here are approximated in only two significant digits and the uncertainties are not included. See the original paper for the exact measured values or Simone 2019, Section 2.2.5 for a summary.

of cold dark matter), and the remaining 68% of the budget is in the form of vacuum energy (dark energy).

3. Microscopic nature of DM

The dynamical mass can be said to be categorized into baryonic matter (ordinary matter)[8] and non-baryonic matter (DM). There is no a priori reason to assume that DM should be a particle: macroscopic features resulting in the existence of DM do not pinpoint to the nature of DM being microscopic. However, to be able to fit it into the paradigm of Standard Model (SM), one has to assume that DM is a particle, alas a very weakly-interacting one.

Let us first consider DM being macroscopic. It is still possible, though very unlikely, that DM is a baryonic astrophysical object yet unseen such as a brown dwarf candidate (Chow 2008, p. 217). In general such objects are called MACHOs (massive astrophysical compact halo objects). Other notable candidates, not really falling in any of the two categories, are primordial black holes. These are black holes that could have been produced at the early stages of the cosmological expansion of the Universe (Frolov et al. 1997, Section 9.8).[9]

Now to consider the microscopic nature of DM, let us list some of the DM acquired properties. For our treatment, we shall assume that a particle candidate for DM must satisfy at least the following fundamental properties (Simone 2019, Section 5.2):

1. Stable, i.e. non-decaying, or with a lifetime longer than the age of the Universe (Bertone 2010, Section 24.2).

2. No electric charge, no color charge, otherwise it would be luminous.

3. Non-collisional, or at least much less collisional than baryons: self-annihilation cross sections must be smaller than the QCD or the weak interaction cross sections.

4. Not *hot* (which is a technical term for not relativistic), since such a scenario is excluded by cosmological models that govern large-scale structure formation (Kravtsov et al. 2012). Alternatively, it should be *cold* (non-relativistic, classical) today, in order to be confined on galactic scales.[10]

5. In the fluid limit, not in the form of a collection of discrete compact objects, which is supported by DM mapping in galaxy surveys (Necib et al. 2018).

Taking these properties at face value, we conclude that the SM of particle physics contains no suitable particle as a candidate for DM. The quarks u, c, t, d, s, b and leptons e, μ, τ are eliminated by property (2) since they feel the electromagnetic interaction. And the particles that are left, neutrinos, are eliminated by property (4) because they are hot. The Higgs boson is eliminated by property (1) because it is not a stable particle. Following this line of thought, we should be looking for a particle which is massive (property 4), and interacts with other particles only via the weak interaction (property 2). This weakly-interacting massive particle is known as WIMP. This is a generic term used for many candidates, namely, the supersymmetric neutralino, Higgs portal scalar, heavy neutrino, lightest Kauza-Klein particle, etc., because many of them share the same

[8] By definition, baryonic matter should only include matter composed of baryons, i.e. quarks, but exclude leptons. But since the leptonic mass makes up for less than 1% of the matter, the term baryonic is synonymous with normal atomic matter for all practical purposes.

[9] For a more recent discussion about the status of primordial black holes in an article, read Sokol 2020.

[10] The not hot and cold DM are to be differentiated, strictly speaking, because there is also the *warm* DM which would be excluded otherwise (see, for example, Bertone 2010, Section 12.2.3).

production mechanism in the early Universe, through the process of thermal freeze-out which is to be discussed in Section 5. The non-WIMP DM candidates, such as the aforementioned primordial black holes, or others such as axions, gravitinos, sterile neutrinos, and superheavy dark matter are produced in specific ways to be studied case-by-case and will not be discussed here.[11]

4. Dynamics of the Universe and Friedmann equations

In this section we will summarize the properties of the Universe and its governing dynamics laying the ground for Section 5. After stating the properties and their consequences, by using Einstein's field equations (EFE) and the conservation of energy-momentum (CEM), we will find the Friedmann equations and the resulting ingredients of our Universe.

On very large scales (distances larger than 100 Mpc[12]):

1. The Universe around us appears to be homogeneous and isotropic, i.e. no observer is special and there are no preferred directions (Misner et al. 1973, Section 27.3).

2. The Universe is expanding at an ever-increasing rate. Furthermore, the expansion of the Universe itself is isotropic. If it were not we would observe large temperature anisotropy in the Cosmic Microwave Background (CMB); although it is known that the microwave background radiation is not perfectly smooth, the deviations (temperature anisotropies) are on the order of 10^{-5} or less. For describing this Universe, the Friedmann-Robertson-Walker (FRW) metric would do the job

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2 \left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta d\mathrm{d}\phi^2\right], \tag{3}$$

   where the curvature parameter k can take three values, $+1$ (positive spatial curvature), 0 (zero spatial curvature), -1 (negative spatial curvature); $a(t)$ is a time-dependent parameter, commonly known as the scale factor, and the spatial coordinates are the usual spherical coordinates.

3. Additionally, by taking the large-scale perspective, galaxies can be treated as particles of a gas that fills the Universe. In other words, the Universe can be treated as a perfect fluid that is characterized by the proportionality relation between pressure p and energy density ρ

$$p = w\rho, \tag{4}$$

   where w is constant in time denoting the type of the cosmological fluid, and its energy-momentum tensor $T_{\mu\nu} = (p+\rho)u_\mu u_\nu - pg_{\mu\nu}$, where u_μ is the four-velocity. Since a tensor can be specified in any reference frame, we will choose the reference frame to be a comoving frame with the fluid, where $u_\mu = (1,0,0,0)$; in which case

$$T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p). \tag{5}$$

The conservation of the energy-momentum, CEM, is expressed by the equation

$$\nabla_\mu T^{\mu\nu} = 0. \tag{6a}$$

The governing gravitational laws, EFE, where the spacetime metric $g_{\mu\nu}$ and its corresponding Ricci tensor $R_{\mu\nu}$ and Ricci scalar R are related to the energy content expressed through the energy-momentum tensor $T_{\mu\nu}$, are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \tag{6b}$$

[11] See, for example, Ellis 2003, Section 2 or Bertone 2010, Section 10, for an account of some of these candidates.
[12] To illustrate, 1 pc (parsec), which is a customary cosmological unit of distance, is approximately 3 light years.

where G is the Newton's gravitational constant; $G_{\mu\nu}$ is the Einstein's tensor and Λ is the cosmological constant term.[13]

The EFE, Eq. (6b), and the CEM, Eq. (6a), are not completely independent of each other. Using the FRW metric, Eq. (3), and the perfect fluid energy-momentum tensor, Eq. (5), the (00) term of EFE gives our first equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}, \tag{7a}$$

and the (0) term of CEM gives us our second equation,

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{7b}$$

where the denoted

$$H \equiv \frac{\dot{a}}{a}, \tag{8}$$

represents the rate of expansion, which is called the Hubble parameter. Consequently, these two equations constitute a complete system of equations, the Friedmann equations, which govern our Universe (Carroll 2014, Section 8.3).

Noticing the appearance of (energy) density in the Friedmann equations we can certainly write the first equation also as a sum of densities, as is customary. Rewriting Eq. (7a),

$$1 = \frac{8\pi G}{3H^2}\left(\rho + \frac{\Lambda}{8\pi G} - \frac{3k}{8\pi G a^2}\right),$$

we motivationally note the three density terms in brackets as

$$\rho = \rho_{\text{matter}} + \rho_{\text{radiation}}, \qquad \rho_{\Lambda} = \frac{\Lambda}{8\pi G}, \qquad \rho_{\text{curvature}} = \frac{3k}{8\pi G a^2},$$

where ρ_{matter} is the matter energy density, $\rho_{\text{radiation}}$ is the radiation energy density, ρ_{Λ} is the cosmological constant energy density, and $\rho_{\text{curvature}}$ is the curvature energy density. The term before the brackets is a measurable quantity, it is denoted by $\rho_c = \frac{3H^2}{8\pi G}$ and it is called the critical density. The energy density today of each component is normalized to the critical density to provide the corresponding "Omega parameter" for matter, radiation, curvature, and cosmological constant. Thus, Eq. (7a) can finally be written as a sum rule in terms of Omega parameters,

$$\Omega_{\text{matter}} + \Omega_{\text{radiation}} + \Omega_{\Lambda} + \Omega_{\text{curvature}} = 1, \tag{9}$$

where

$$\Omega_{\text{matter}} \equiv \frac{\rho_{\text{matter}}}{\rho_{\text{c}}}, \qquad \Omega_{\text{radiation}} \equiv \frac{\rho_{\text{radiation}}}{\rho_{\text{c}}}, \qquad \Omega_{\Lambda} \equiv \frac{\rho_{\Lambda}}{\rho_{\text{c}}}, \qquad \Omega_{\text{curvature}} \equiv \frac{\rho_{\text{curvature}}}{\rho_{\text{c}}}.$$

Following our consequent constraint, Eq. (9), the energy densities are than made to fit observational data, as discussed in Section 2.3. For emphasis we again note that DM comes into the picture a priori by virtue of the hypothesis that it exists and dimensionally it is a part of ρ_{matter}.

The second equation, Eq. (7b), can be solved for ρ as a function of a, using Eq. (4),

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = 0 \qquad \rightarrow \qquad \rho(a) \sim a^{-3(1+\omega)},$$

and since $\omega = 0$ for matter, $\omega = 1/3$ for radiation, and $\omega = -1$ for vacuum energy (Baumann n.d., pp. 18-20), we respectively get

$$\rho(a) \sim a^{-3}, \qquad \rho(a) \sim a^{-4}, \qquad \rho(a) \sim a^{0}.$$

This result shows that for different kinds of cosmological fluids the energy density scales down with the expansion of the Universe with different powers of the scale factor. Once again, to emphasize, DM is part of matter.

[13] For an account of Einstein's field equations and his cosmological constant, see, Carroll 2014, Section 4.2 and 4.5.

5. DM particles as thermal relics

In this section we will consider the WIMP candidate for DM, assuming that DM is a relic from the so-called freeze-out epoch of the Universe.

We can extract a well-known statement from the property of an expanding Universe discussed in Section 4. Extrapolating back from today, we conclude that at early times the density ρ of the Universe was very high (i.e. the distances between objects governed by $a(t)$ were extremely small). Then, from the equation of state, Eq. (4), it follows that the pressure p was very high, and hence by thermodynamics, this resulted in high temperature T. In other words, at early times, the Universe was hot and dense and as it expanded it became cooler.

At early times then, the thermodynamical properties of the Universe were determined by local equilibrium dynamics. Afterwards, the Universe evolved to be in non-equilibrium. Departures from equilibrium are responsible for most of the cosmological abundances we observe today; without such departures, the past history of the Universe would not matter for the present observations, which we know is not the case. The expansion affected the rate of interactions for all the constituents in the Universe. The rate of particle interactions can be defined as

$$\Gamma \equiv n\langle \sigma v \rangle, \tag{10}$$

where n is the number density of particles and $\langle \sigma v \rangle$ is the thermally averaged interaction cross section times the velocity of particles. The key to understanding the thermal history of the Universe is the comparison between the rate of interactions Γ, Eq. (10), and the rate of expansion H, defined in Eq. (8), which we rewrite for convenience,

$$H \equiv \frac{\dot{a}}{a}.$$

As the Universe cools, the rate of interactions typically decreases faster than the expansion rate and at a certain point, the particles decouple from the thermal bath (Baumann n.d., p. 36). For example, the strength of weak interactions decreases as the temperature of the Universe drops, namely, $\sigma \sim T^2$ (Baumann n.d., p. 39). Summarizing, $\Gamma \gtrsim H$ produces coupled particles and $\Gamma \lesssim H$ produces decoupled particles (Kolb et al. 1990, p. 115). Different particle species have different interaction rates and so they decouple at different times. The decoupled particles remain as relics for all future times. For example, CMB measurements yield photons as relics; DM relics can be understood analogously. Furthermore, massive particles are also expected to freeze-out, i.e. at a certain point their relic density becomes constant, as can be seen in Fig. 3. This partially confirms WIMPs being great candidates.

An ideal fluid in non-equilibrium, which is what interests us, can be described by the equation, which can be written as (Kolb et al. 1990, pp. 116-120)

$$\hat{\mathbf{L}}[f] = \mathbf{C}[f],$$

where $\mathbf{C}$ is the collision operator (governing the interaction in question) and $\hat{\mathbf{L}}$ is the Liouville operator (describing the evolution of a phase space volume); both being functions of the phase space density f. The LHS, for a generic species of particles X can be written as

$$\hat{\mathbf{L}} = \frac{\mathrm{d}n_X}{\mathrm{d}t} + 3\frac{\dot{a}}{a}n_X = \frac{1}{a^3}\frac{\mathrm{d}(n_X a^3)}{\mathrm{d}t}, \tag{11a}$$

where the number density is given by

$$n_i = \frac{g(i)}{(2\pi)^3}\int f \,\mathrm{d}^3p,$$

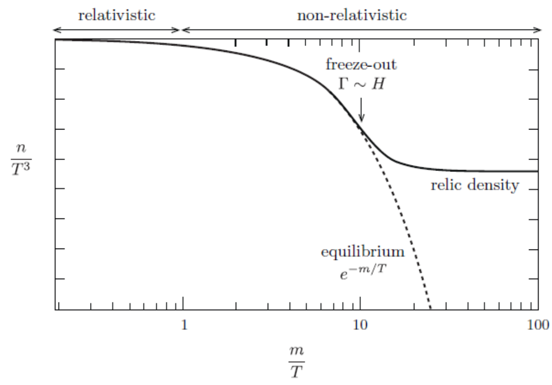

Figure 3. A schematic illustration of particle freeze-out denoting that the freeze-out happens around $M/T = x_f \approx 10$, which means that after that point, the relic density becomes constant. Taken from Baumann n.d., p. 38.

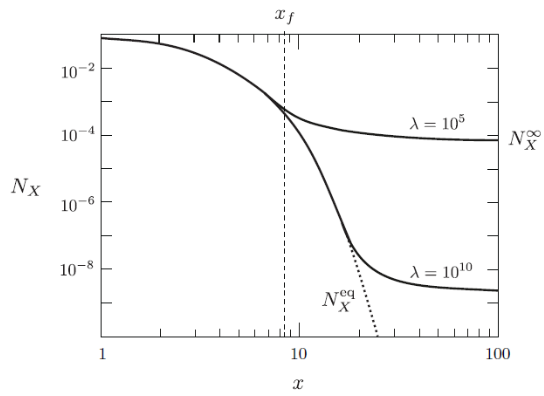

Figure 4. The abundance of dark matter particles as the temerature drops below the mass, denoting the solution of Eq. (15). Taken from Baumann n.d., p. 57.

where g denotes the number of degrees of freedom, peculiar to the kind of particle in question. To be able to write the RHS, we have to know what kind of interaction is taking place. We assume that a massive DM particle X and its antiparticle $\bar{X}$ annihilate producing two light (essentially massless) particles l and $\bar{l}$, i.e.

$$X + \bar{X} \leftrightarrow l + \bar{l} \qquad \equiv \qquad \mathrm{DM} + \mathrm{DM} \ \leftrightarrow \ \mathrm{SM} + \mathrm{SM},$$

where SM stands for standard model particles. Then the RHS can be written as[14]

$$\mathbf{C} = -\langle \sigma v \rangle \left[n_X n_{\bar{X}} - \left(\frac{n_X n_{\bar{X}}}{n_l n_{\bar{l}}} \right)_{\text{eq}} n_l n_{\bar{l}} \right], \tag{12}$$

where the quantity in round brackets results from the condition that the collision term must vanish in equilibrium.

Now let us compile our results. Assuming that $n_l = n_{\bar{l}} = n_l^{\text{eq}}$ and $n_X = n_{\bar{X}}$ and introducing a new variable $N_X \equiv n_X/s$ where s is the entropy density given by S/a^3, and equating Eq. (11a) with Eq. (12), we get the Boltzmann equation in the form

$$\frac{dN_X}{\mathrm{d}t} = -s \langle \sigma v \rangle \left[N_X^2 - (N_X^2)_{\text{eq}} \right]. \tag{13}$$

Changing variables,

$$x = \frac{M_X}{T} \qquad \Rightarrow \qquad \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{M_X}{T}\right) = -T\frac{\mathrm{d}T}{\mathrm{d}t}x \approx -\frac{\dot{T}}{T}x = Hx, \tag{14}$$

where we have used the fact that $T \sim a^{-1}$, we obtain

$$\frac{\mathrm{d}N_X}{\mathrm{d}x} = -\frac{\lambda}{x^2}\left[N_X^2 - (N_X^2)_{\text{eq}} \right], \tag{15}$$

where λ, having absorbed $s \langle \sigma v \rangle$, can be treated as a constant.[15]

Even for constant λ, there are no analytic solutions to Eq. (15). Fig. 4 shows the result of a numerical solution for two different values of λ. At very high temperatures, that is, $x < 1$, by Eq. (14) we have $N_X \approx N_X^{\text{eq}} \approxeq 1$. At low temperatures, $x \gg 1$, the equilibrium abundance becomes

[14] The rest of this section closely follows Baumann n.d., Section 3.3.1 and 3.3.2.

[15] For an explicit form of λ and its constituents, see Baumann n.d., compare Section 3.3.2 and 3.2.3.

exponentially suppressed, $N_X^{\rm eq} \sim e^{-x}$, that is, the particles will become so rare that they will not be able to maintain the equilibrium abundance. Numerically, we find that freeze-out happens at about $x_f \sim 10$, as can be seen in Fig. 4.

The final relic abundance, $N_X^\infty = N_X$ at $x \to \infty$, determines the freeze-out density of DM. Let us estimate its magnitude as a function of λ. After freeze-out, N_X will be much larger than $N_X^{\rm eq}$ (see Fig 4). That is, at late times, we can drop $N_x^{\rm eq}$ from the Boltzmann equation and integrate,

$$\frac{\mathrm{d}N_X}{\mathrm{d}x} \approxeq -\frac{\lambda N_X^2}{x^2} \qquad \to \qquad \frac{1}{N_X^\infty} - \frac{1}{N_X^f} = \frac{\lambda}{x_f},$$

which was obtained by integrating from x_f to $x = \infty$ and we denoted $N_X^f \equiv N_X(x_f)$. Typically, $N_X^f \gg N_X^\infty$, so using this approximation,

$$N_X^\infty \approxeq \frac{x_f}{\lambda}, \tag{16}$$

where we can finally get a feeling for the relic abundance. Eq. (16) predicts that the freeze-out abundance N_X^∞ decreases as the interaction rate λ increases. In other words, larger interactions maintain equilibrium longer. Remembering that λ contains the interaction cross section—compare Eq. (13) to Eq. (15)—we note that having a result of the DM relic abundance enables us to find the kind of interaction in question, hence finding out what is the kind of particles that make up DM, as shown below.

Finally, it remains to relate the freeze-out abundance of DM relics to the DM density today $\Omega_X \equiv \frac{\rho_{X,0}}{\rho_{c,0}}$, which proportionally turns out to be (Baumann n.d., p. 58),

$$\Omega_X \sim \frac{x_f}{g(M_X)\,\langle\sigma v\rangle}.$$

This reproduces the observed DM density if $\sqrt{\langle\sigma v\rangle} \sim 0.1\sqrt{G_F}$, where G_F is the Fermi coupling constant. This cross section is characteristic of the weak interaction. In other words, a thermal relic with a weak interaction cross section gives the right dark matter abundance. This is known as the WIMP miracle.

6. Searches for DM on Earth

There exist several ways of searching for DM. But as we have already emphasized in Section 3, all searches must rely on the interaction of DM particles with matter (SM particles); which means they only work if DM particles participate in other interactions beside the gravitational interaction (see Fig. 5). The DM search still goes on, but so far DM has not been detected by any experiment. The allowed mass region of DM extends from a few eV to around five solar masses (Particle Data Group et al. 2020, Section 27.2). In this section we will qualitatively discuss some experimental results.

6.1 Direct detection

Direct detection experiments aim at detecting the fraction of WIMPs, constantly flowing through the Earth, that elastically scatter off nuclei in underground detectors through the measurement of nuclear recoils (Bertone 2010, p. 10). The recoil energy of the scattered nucleus can be measured and can signal the occurrence of a DM particle scattering by. For example, for a DM particle with mass $m_X = 100$ GeV scattering off a ^{131}Xe nucleus, we get $E_{\rm recoil} \approx 22$ keV (Simone 2019, p. 23). This shows that the recoil energies are in the keV range. XENON Collaboration et al. 2018 published results in these kind of measurements. Even though they have not detected DM particles, their results have added new constraints. Their measurements, for WIMP-nucleon spin-independent elastic scatter cross-section, for WIMP masses above 6 GeV$/c^2$, exclude the range of parameters $\sigma_{X,\rm min} = 4.1 \times 10^{-47}$ cm^2 at $m_X = 30$ GeV$/c^2$ with a 90% confidence level (see Fig. 6).

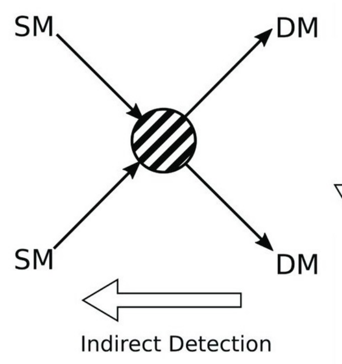

Figure 5. A schematic illustration denoting a unified picture of the direct and indirect methods to search for DM. The direct detection means detecting WIMPs via their interaction with SM particles. The indirect detection means only detecting properties of SM particles which would become uniquely manifest via their interaction with DM. Adapted from Giagu 2019, p. 2.

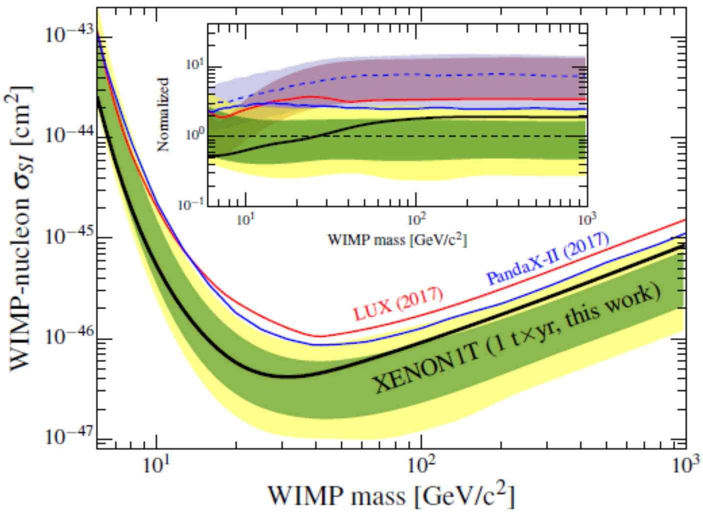

Figure 6. An example of the direct detection method through the measurement of nuclear recoils which enable to graph the WIMP-nucleon cross-section in terms of the WIMP mass. Although there is no detection of DM particles, these results have added new constraints: there are no DM in this range of parameters. See the original reference for more details. Taken from XENON Collaboration et al. 2018, p. 7.

6.2 Indirect detection

The indirect searches for DM are based on identifying excesses in fluxes of gamma rays/cosmic rays with respect to their presumed astrophysical backgrounds (Simone 2019, p. 24). A schematic chain of events leading from DM annihilation to observable fluxes is

$$XX \quad\rightarrow\quad \text{SM SM} \quad\rightarrow\quad \text{stable species} \quad\rightarrow\quad \text{fluxes at Earth}$$

The most promising sources of DM annihilations are the places where DM is expected to be the densest, for example, the galactic center, the inner halo of our Galaxy, nearby galaxies dominated by DM etc. However, detection from these sources may be difficult, astrophysically speaking. This means that maybe the best places for detection might not be the sources richest with DM but those with well-identified backgrounds, i.e. those with favorable signal for our measurement facilities. Such measurements were conducted by Fermi-LAT Collaboration et al. 2015. They also have added new constraints. Their measurements exclude the thermal relic annihilation cross section for WIMPs with $m_X \lesssim 100$ GeV annihilating through the quark and τ-lepton channels (the significant low-energy bound being 500 MeV).

7. Conclusion and prospects

Dark matter is still an unknown component of our Universe. Though it is accepted that it is a necessary component of the standard cosmological model needed to describe the large-scale structure of our Universe, it is still not established how it should concord with the standard model of particle physics. The searches have still not covered the whole allowed mass region, so WIMPs remain viable candidates. The searches mentioned above ought to be concerned with masses above $\mathcal{O}$(GeV). Another strategy being considered at the present times is to search for scattering off bound electrons, allowing to probe masses down to $\mathcal{O}$(MeV).

The other possibility, that of dark matter being non-microscopic, leads to the pursuit of candidates such as primordial black holes or the theory of modified Newtonian dynamics (MOND).

Finally, dark matter may turn out to be a consequence of a more fundamental theory, outside of the existing paradigm, which could encompass an unforeseen explanation.

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