MAGNETOHYDRODYNAMICS

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Magnetohydrodynamics studies the behavior of an electrically conductive fluid in a magnetic field. The first part of the article presents the basics of magnetohydrodynamics. Noncentral collisions of heavy particles cause the formation of a very strong magnetic field in the quark-gluon plasma. With some assumptions, one can calculate this magnetic field. Finally, the result of how the magnetic field behaves in a vacuum and in the presence of a conductive medium is shown.

MAGNETOHIDRODINAMIKA

Magnetohidrodinamika preučuje obnašanje električno prevodne tekočine v magnetnem polju. V prvem delu članka so predstavljene osnove magnetohidrodinamike. Necentralni trki težkih delcev povzročijo nastanek zelo močnega magnetnega polja v kvark gluonski plazmi. Z nekaterimi predpostavkami lahko izračunamo to magnetno polje. Na koncu je še prikazan rezultat, kako se obnaša magnetno polje v vakuumu in ob prisotnosti prevodnega medija.

1. Introduction

Magnetohydrodynamics (MHD) is the description of the behavior of plasma, or in general, any electrically conducting fluid, when electromagnetic fields are dynamical. The fundamental concept of MHD is that magnetic fields can initiate currents in moving conductive liquids, which in turn creates forces on the fluid and changes the magnetic field itself. A set of equations describing MHD is a combination of Navier-Stokes fluid dynamics equations and Maxwell's elementary equations of electromagnetism.

When the Universe was just a few microseconds old, it consisted of a hot soup of elementary particles, called quarks and gluons. At that time, the temperature and the density were too high for protons, neutrons or any hadrons to form. A few microseconds later, those particles began cooling to form protons and neutrons. Relativistic hydrodynamics was proven to be an excellent tool to model the dynamical evolution of the quark-gluon plasma (QGP) and investigate its properties. The inclusion of small dissipative effects allowed a closer match with the experimental observations, especially with the anisotropic spatial distribution of the detected particles. Despite its great success, it has been found that one of the fundamental requirements for the use of hydrodynamics may not be fully fulfilled, especially in the early times after a collision. This was solved by adding the missing component: the magnetic field. The use of MHD in heavy ion collisions opens a new possibility to explore the electromagnetic properties of the QGP [1], [2], [3].

2. Magnetohydrodynamics

2.1 Ideal magnetohydrodynamics

Ideal MHD is a limiting regime, which means neglecting all dissipative processes (finite viscosity, electrical resistivity, and thermal conductivity) and assuming isotropic pressure.

First equation of ideal MHD is mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0, \tag{1}$$

where ρ is plasma mass density and $\vec{v}$ is plasma velocity. It states that matter is neither created or destroyed. The partial derivative refers to the change in density at a single point in space and the divergence of the mass flux says how much plasma goes in and out of the region.
The second equation is the equation of motion of an element of fluid or the Euler equation in the presence of the Lorentz force

$$\rho\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\,\vec{v}\right]=-\nabla P+\vec{j}\times\vec{B}, \tag{2}$$

where P is thermodynamics pressure, $\vec{B}$ is magnetic field and $\vec{j}$ is the electric current density.
The third equation in the simplest adiabatic case is the energy equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{\rho^\gamma}\right)=0, \tag{3}$$

where γ is the ratio of specific heats $\frac{C_p}{C_v}$, normally taken as $\frac{5}{3}$. In pure hydrogen plasma, the equation for preassure is

$$P=2\frac{k_B}{m_p}\rho T, \tag{4}$$

where m_p is mass of a proton and k_B is Boltzmann's constant.
The equation for the magnetic field is derived from Maxwell's equation, starting with Ohm's law

$$\vec{j}=\sigma\vec{E}' \tag{5}$$

where σ electrical conductivity and $\vec{E}'$ is the electric field experienced by the plasma element in its rest frame. When the plasma is moving (concerning the external magnetic field) at the velocity $\vec{v}$, the Lorentz transformation is

$$\vec{E}'=\vec{E}+\vec{v}\times\vec{B} \tag{6}$$

In the case of perfect conductivity, $\sigma\to\infty$, the equation (6) is rewritten as

$$\vec{E}=-\vec{v}\times\vec{B}. \tag{7}$$

By calculating the curl of the electric field $\vec{E}$ and using Faraday's law

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \tag{8}$$

one can exclude the electric field and get the fourth MHD equation – induction equation

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times\left(\vec{v}\times\vec{B}\right), \tag{9}$$

which describes the phenomenon of the magnetic dynamo.
To close the set of MHD equations, the current density j is expressed through the magnetic field by considering the Maxwell's equation,

$$\nabla\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{j}. \tag{10}$$

From Equation (7), the electric field magnitude can be estimated as $E\sim V_0B$, where V_0 is a characteristic speed of the process. When the process is non-relativistic, the first term in Equation (10) is much bigger than the second, and one gets

$$\vec{j}=\frac{1}{\mu_0}\nabla\times\vec{B}. \tag{11}$$

The magnetic field $\vec{B}$ must also satisfy the condition $\nabla\cdot\vec{B}=0$, which means that magnetic monopoles do not exist. For more details on the theory and derivations, refer to references [1], [4].

2.2 Dissipative magnetohydrodynamics

In the case of dissipative MHD, the set of equations becomes

$$\rho\frac{\partial\vec{v}}{\partial t}+\rho\left(\vec{v}\cdot\nabla\right)\vec{v}=-\nabla P-\frac{1}{\mu_0}\vec{B}\times\nabla\times\vec{B}+\vec{\mathcal{F}}, \tag{12}$$

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times\left(\vec{v}\times\vec{B}\right)+\eta\nabla^2\vec{B}, \tag{13}$$

$$\frac{\partial\rho}{\partial t}+\nabla\left(\rho\vec{v}\right)=0, \tag{14}$$

$$\frac{\rho^\gamma}{\gamma-1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p}{\rho^\gamma}\right)=-\mathcal{L}. \tag{15}$$

The term $\mathcal{L}$ is energy loss/gain function and the parameter η is the magnetic diffusivity, connected with the electrical conductivity σ,

$$\eta=\frac{1}{\mu_0\sigma}. \tag{16}$$

The term $\vec{\mathcal{F}}$ is an external force acting on a unit of volume of the plasma. For example, the external force for the gravity and the viscosity is

$$\vec{\mathcal{F}}=-\rho\vec{g}+\nu\rho\left[\nabla^2\vec{v}+\frac{1}{3}\nabla(\nabla\cdot\vec{v})\right], \tag{17}$$

where $\vec{g}$ is gravitational acceleration and ν is coefficient of kinematic viscosity.

2.3 Relativistic magnetohydrodynamics

Relativistic fluid dynamics plays an important role in understanding the dynamics of ultrarelativistic heavy ion collisions. In this article, the metric signature of $\mathrm{g}_{\mu\nu}$ = diag(-, +, +, +) is used, along with natural units where $c=1$.

2.3.1 Maxwell's equations

The electromagnetic field is completely described by the Faraday electromagnetic tensor field $F^{\mu\nu}$, obeying Maxwell's equations:

$$\nabla_\nu F^{\mu\nu}=4\pi J^\mu, \tag{18}$$

$$\nabla^*_\nu F^{\mu\nu}=0, \tag{19}$$

where ∇_ν is the covariant derivative, J^μ is the charge four-vector current and ${}^*F^{\mu\nu}$ is the dual electromagnetic field tensor defined through the relation

$${}^*F^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\gamma\rho}F_{\gamma\rho}, \tag{20}$$

where $\epsilon_{\mu\nu\gamma\rho}$ is the Levi-Civita tensor.
Magnetic induction field B^α and electric field E^α are expressed with four-velocity u

$$B^\alpha={}^*F^{\alpha\beta}u_\beta, \tag{21}$$

$$E^\alpha=F^{\alpha\beta}u_\beta. \tag{22}$$

The charge current four-vector J^μ can be expressed as

$$J^\mu=qu^\mu+\sigma F^{\mu\nu}u_\nu, \tag{23}$$

where q is the proper charge density and σ is the electric conductivity. If the fluid is a perfect conductor (ideal MHD condition) then $\sigma \to \infty$. In order to keep the current finite it is necesary to impose $F^{\mu\nu}u_\nu = 0$. In this case the electromagnetic tensor can be written in terms of the magnetic field b^μ

$$F^{\nu\sigma} = \epsilon^{\alpha\mu\nu\sigma} b_\alpha u_\mu \tag{24}$$

and the dual of this expression is

$$^*F^{\mu\nu} = b^\mu u^\nu - b^\nu u^\mu. \tag{25}$$

As a result, the Maxwell's equations become

$$\nabla^*_\nu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu \left(\sqrt{-g} \left(b^\mu u^\nu - b^\nu u^\mu \right) \right) = 0, \tag{26}$$

where $\sqrt{-g}$ is determinant of the metric $g_{\mu\nu}$.

2.3.2 Conservative equations

Equations for the rest mass density ρ, the specific internal energy ϵ and for the three-velocity v^i can be computed from the conservation of the barion number

$$\nabla_\nu \left(\rho u^\nu \right) = 0, \tag{27}$$

from the conservation of the energy-momentum tensor

$$\nabla_\nu T^{\mu\nu} = 0, \tag{28}$$

and from an equation of state relating the pressure p to the rest mass density ρ and to the specific internal energy ϵ.

The energy-momentum tensor $\mathrm{T}^{\mu\nu}$ can be splitted in two parts: one for the fluid $\mathrm{T}^{\mu\nu}_{\text{fluid}}$ and one for the electromagnetic field $\mathrm{T}^{\mu\nu}_{\text{em}}$. For the perfect fluid the energy-momentum tensor is

$$T^{\mu\nu}_{\text{fluid}} = \rho h u^\mu u^\nu + p g^{\mu\nu}, \qquad h = 1 + \epsilon + \frac{p}{\rho}, \tag{29}$$

where h is the specific relativistic enthalpy and the energy-momentum tensor for electromagnetic field is

$$T^{\mu\nu}_{\text{em}} = \frac{1}{4\pi} \left(F^\mu{}_\gamma F^{\nu\gamma} - \frac{1}{4} g^{\mu\nu} F_{\gamma\delta} F^{\gamma\delta} \right). \tag{30}$$

By using the definition of magnetic field b, one can write energy-momentum tensor for the electromagnetic field as

$$T^{\mu\nu}_{\text{em}} = \left(u^\mu u^\nu + \frac{1}{2} g^{\mu\nu} \right) b^2 - b^\mu b^\nu. \tag{31}$$

The total energy-tensor is then

$$T^{\mu\nu} = \left(\rho h + b^2 \right) u^\mu u^\nu + \left(p + \frac{1}{2} b^2 \right) g^{\mu\nu} - b^\mu b^\nu, \tag{32}$$

As one can see above, Maxwell's equations and conservative equations concern the mutual interaction of fluid flow and magnetic field [5], [6].

3. Magnetohydrodynamics in heavy ion collisions

QGP is created in Heavy Ion Collisions. The collision of two ultrarelativistic heavy ions produces a small amount of a new phase matter called QGP. This extremely hot high-density matter consists of deconfined quarks, antiquarks, and gluons that are strongly coupled to each other and form a collective medium with properties of a fluid. Because the lifetime of QGP before it breaks into hadrons is about 10^{-24}s, detectors can only measure the particles emerging from QGP but not the medium itself, which makes the analysis of QGP much more complex. Its properties are mostly determined by studying thermal photons, jet quenching, and elliptic flow. In these collisions, there should be large electromagnetic fields present that affect the evolution of the QGP. The study of QGP in the laboratory is also the only experimentally accessible approach to improve the understanding of the early Universe when the Universe was only a few microseconds old [2], [7].
At present time the properties of quark-gluon plasma are studied in two heavy-ion colliders: Large Hadron Collider (LHC) in Cern and Brookhaven National Laboratory's Relativistic Heavy Ion Collider (RHIC) [1].

3.1 Quark gluon plasma

Quarks and antiquarks are elementary particles that create mesons and baryons, which are bound by gluons. Particles containing quarks in antiquarks are called hadrons. The theory that describes the strong interactions of the quarks and gluons is known as Quantum Chromo Dynamics (QCD). The strong force is described with three color charges and force mediators gluons, which also carry the color charge. Therefore it can be expected that the structure of the phase diagram of matter derived from QCD is even richer than what is known till now.
There are six different flavors of quarks along with their antiquarks. The strength of the strong force decreases with decreasing distance between quarks. This behavior of the strong force is opposite to electromagnetism and gravity. One can understand it by considering gluon analogously to an elastic string. The coupling between two quarks increases with distance in such a way that if a quark and anti-quark pair are separated, then the creation of a new quark anti-quark pair is more favorable than the isolation of the quark and the anti-quark. These phenomena are known as confinement i.e. quarks are confined to hadrons and cannot exist freely in nature.
The coupling weakens on decreasing the separation between a quark and antiquark pair, so quarks effectively move freely. This phenomenon is known as asymptotic freedom and is the deconfined state of quarks in which they can move in a volume much larger than the volume of a nucleon. Deconfined quarks have never been seen at normal temperature and energy density. If the energy density and temperature of the system are increased sufficiently, normal nuclear matter (protons and neutrons) will deconfine into a sea of free quarks and gluons. This happens above some critical temperature, T_c, calculated to be approximately 175 MeV according to lattice QCD. This set of free quarks and gluons is known as QGP [2], [8], [9].

3.2 Collision

Only the final collision products of outgoing particles are measurable, but the QGP stage is fascinating. The evolution of heavy ion collision is easily understandable by dividing it into several stages. These stages are:

- **Initial state**: Two incoming accelerating nuclei will appear as two flat blobs approaching each other due to Lorentz contraction. This can be explained by Gubser's flow and it is briefly described below.

- **Thermalization**: Two flat blobs collide and most of the interactions of quarks, antiquarks, and gluons, partons for short, are soft. This means that they lose some energy but they involve little transverse momentum transfer (their initial trajectory does not change much). The system approaches thermal equilibrium.

- **QGP**: When the incident nuclei collide, a small fraction of the incident partons is involved in hard perturbative interactions. This leads to the production of partons with high transverse momentum.

- **Hadronization**: After the QGP stage, the energy density and temperature of the matter decrease continuously until the interaction between the partons is strong enough to transform the partons into hadrons – a jet of many particles in the initial quark's direction of travel.

- **Hadron gas**: Hadrons exhibit collective behavior, even though they are weakly coupled. The system is in equilibrium and behaves like a dilute gas.

- **Freeze-out**: When the energy density drops below the energy density within an individual hadron (around 500 $\mathrm{MeV/fm^3}$) the local thermalization breaks and the QGP falls apart into a dense mist of hadrons. The interaction between the hadrons becomes weaker, and the mean free path of the hadrons is larger than the size of the system.

For more details, see [2], [8].

3.3 Impact parameter

The collisions are characterized by impact parameter b, representing the transverse distance between the centers of masses of the two nuclei. When the impact parameter is small the collision is almost central. In this case, high transverse momentum partons have longer paths inside the QGP, on average they lose more energy and tend to have the maximum energy density. A peripheral collision with a large impact parameter has a small overlap zone. The high transverse momentum parton emerges after losing less energy and tends to have lower energy density. In a non-central collision, the process of production of the QGP is the same in the overlap region. The parts of the incident nuclei that do not collide are referred to as spectators, and at very early times, they create a magnetic field in the collision zone. In the extreme limit the heavy ions miss each other but the Lorentz-contracted disc of electromagnetic fields around them still interacts [2], [9], [10].

3.4 Electromagnetic field

It is very well known what these collisions look like and the presence of an early-time magnetic field can have observable consequences on the motion of the final-state charged particles seen in detectors. One can investigate the origin of induction of electric currents carried by the charged quarks and antiquarks in the QGP and, later, by the charged hadrons. The origin of these charged currents is twofold. Firstly, there is an electromagnetic field due to the moving spectators. These moving charged spectators generate a magnetic field and when they move away from the QGP their electromagnetic field drops. Because of that, electromagnetic field changes in time. The changing magnetic field results in an electric field because of Faraday's law, and this, in turn, produces an electric current in the conducting medium. Secondly, the Lorentz force, $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$, triggers the Hall effect. This force is perpendicular to the longitudinal speed and the magnetic field. The Hall effect occurs because charged particles of different signs are directed to opposite sides, which in turn generates another electric field. The net electric current is the sum of that due to Faraday and due to Hall.

To get the analytical solution for electromagnetic field in heavy-ion collisions, firstly, one needs an analytic solution for the hydrodynamic expansion of the QGP in the absence of any electric currents. This can be done by using Gubser's model [11]. Gubser's model describes symmetric systems that expand azimuthally in the transverse plane and boost invariant longitudinal expansion ($SO(1,1)\times SO(3)\times Z_2$). This model gives the local fluid temperature and the four-velocity u_μ, which describes the fluid velocity of QGP. The analytic solution for relativistic viscous hydrodynamics for a conformal fluid with the shear viscosity to entropy density ratio is given by $\eta/s = 1/4\pi$.
To obtain the velocity $\vec{v}$, which contains both the expansion because of Gubser's model and the contribution of the electromagnetic field, one can do this by doing a Lorentz boost[1] to the local fluid rest frame in which applies $u_\mu = 0$. To get the velocity due to the electromagnetic field $\vec{v'}$, the equation of motion needs to be solved

$$m\frac{\mathrm{d}\vec{v'}}{\mathrm{d}t} = q\vec{v'}\times\vec{B'} + q\vec{E'} - \mu m\vec{v'} = 0, \tag{33}$$

where μ is drag coefficient[2] and that means that the last term describes the drag force on a fluid element with mass m on which electromagnetic force is being exerted. The drag coefficient μ is chosen from $N = 4$ supersymmetric Yang-Mills (SYM) theory [12] and it is precisely known only for heavy quarks

$$\mu m = \frac{\pi\sqrt{\lambda}}{2}T^2, \tag{34}$$

where $\lambda = g^2 N_c$ is the 't Hooft coupling, g is gauge coupling and N_c is the number of colors. To ease the calculation of magnetic fields, one can estimate that μm is a constant at $\mathrm{T}=\frac{3}{2}\,\mathrm{T}_c$, where $\mathrm{T}_c \sim 170$ MeV is the crossover temperature to hadrons.
Equation (33) can be solved for up quarks and anti down quarks and the result is that one gets positively charged particles. Two found velocities can be averaged to get the final velocity. For negatively charged particles the down and anti up quarks are used. The particle densities for u and d quarks and antiquarks are all the same and because of that one can neglect any chemical potentials for baryon number or isospin. The negative particles will be characterized by $-\vec{v}'$. When one solves this equation it can be boosted back to the original frame with including u_μ and obtaining final four-velocity V_μ. The four-velocity V^μ includes both the velocity of the positively (or negatively) charged particles due to electromagnetic effects and the much larger, charge independent, velocity u of the expanding plasma [8], [10].

3.4.1 Calculations of the electromagnetic field

The magnetic field is produced by the charged ions (spectators) in a non-central collision. Expressions for the electromagnetic field in center of mass frame can be obtained by Maxwell's equations and by using Ohm's law $\vec{j} = \sigma\vec{E}$, where $\sigma = 0.023$ fm^{-1}.
One can start with wave equation

$$\nabla^2\vec{B} - \partial_t^2\vec{B} - \sigma\partial_t\vec{B} = -ev\nabla\times\left[\hat{z}\delta\left(z-vt\right)\delta\left(\vec{x}_\perp - \vec{x'}_\perp\right)\right] \tag{35}$$

Coordinate with the apostrophe denotes the location of the particle that creates the field. As it turns out, solving this equation is a long procedure. It can be solved by using Green's functions

[1] The Lorentz boost is the Lorentz transformation, which doesn't involve rotation. It describes the motion of the space coordinate axes with a constant velocity.

[2] The drag coefficient is a dimensionless quantity in fluid dynamics that determines the drag or resistance of a body in a liquid.

and complex analysis. The result for the magnetic field in the y-direction is

$$eB_y^+(\tau,\chi,x_\perp,\phi) = \alpha\hat{y}\sinh(Y)\left(x_\perp\cos(\phi) - x'_\perp\cos(\phi')\right) \times \frac{\left(\frac{\sigma|\sinh(Y)|}{2}\sqrt{\Delta}+1\right)}{\Delta^{3/2}}e^A, \tag{36}$$

where + sign represents the moving particle in $+z$ direction, $\tau = \sqrt{t^2 - z^2}$ is proper time, $Y = \text{arctanh}(v)$ is rapidity, α is the electromagnetic coupling, $\chi = \text{arctanh}(z/t)$ is pseudorapidity, $x_\perp$ is equivalent of $\sqrt{x^2+y^2}$ in Cartesian coordinates and ϕ is azimuthal angle. A and Δ are defined as follows

$$A = \frac{\sigma}{2}\left(\tau\sinh(Y)\sinh(Y-\chi) - |\sinh(Y)|\sqrt{\Delta}\right), \tag{37}$$

$$\Delta = \tau^2\sinh(Y-\chi) + x_\perp^2 + x_\perp'^2 - 2x_\perp x'_\perp\cos(\phi-\phi'). \tag{38}$$

The same procedure can be used to solve the equation for the electric field:

$$eE_x^+ = eB_y^+\coth(Y-\chi). \tag{39}$$

The other components of the electromagnetic field are irrelevant because B_z=0.

To obtain the total contribution of all particles one needs to integrate over the coordinates indicated with an apostrophe in Equation (36) with some distribution function. In both cases, it can be assumed that the particles were evenly distributed over a sphere with R = 7 fm. In this case b = 7 fm. Using the distribution the limits of $x'_\perp$ integrals can be found. For the participants, one needs to integrate over the part that will overlap in the collision, and for spectators, one needs to integrate over the non-overlapping area [10]. The results of this calculation are shown in Figure 1.

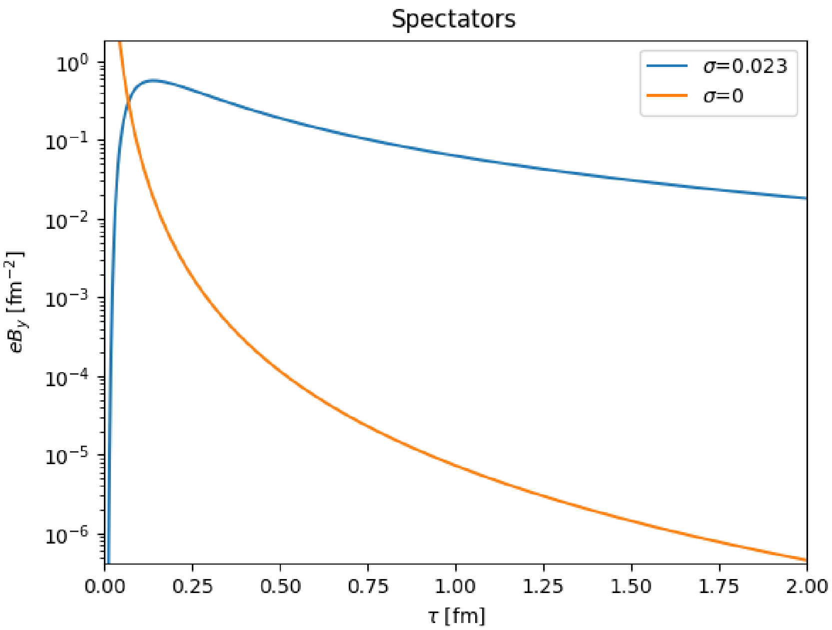

Figure 1. Magnetic field B_y perpendicular to the reaction plane produced by the spectators in a heavy-ion collision at LHC. The blue curve shows how rapidly B_y would decay as the spectators recede if there were no medium present (in a vacuum with $\sigma = 0$) and, the orange curve shows the presence of a conducting medium with $\sigma = 0.023$ fm^{-1} [10].

4. Conclusion

MHD studies the dynamics of electrically conducting fluids. It establishes a coupling between the Navier-Stokes equations for fluid dynamics and Maxwell's elementary equations for electromagnetism. The main concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and influence the magnetic field itself. MHD should play an important role in the phenomenology of relativistic heavy-ion collisions. The collision of two ultrarelativistic heavy ions produces QGP. Electric currents carried by QGP are created by the moving spectators in a non-central collision. With a few assumptions described in Section 3, the electromagnetic field in these heavy-ion collisions can be calculated.

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