

WALKING ON WATER

BOŠTJAN MELINC

Fakulteta za matematiko in fiziko
Univerza v Ljubljani

In this article, the main techniques used by creatures to move on the water surface and a physical insight into them are reviewed. Firstly, some basic equations, which are important for understanding the physics of walking on water, are derived. Secondly, attention is given to the ways used by arthropods to stand and walk on water. The main part of the article is dedicated to large water-walkers with an emphasis on a hydrodynamic model for a water-walking basilisk lizard. At the end, the reasons why a human is not able to walk on water are discussed, and some techniques that would enable humans to walk on water are described.

HOJA PO VODI

Članek podaja fizikalni vpogled v tehnike, ki jih živali uporabljajo za hojo po vodi. Sprva se osredotoči na izpeljavo in razlago osnovnih enačb, potrebnih za opis gibanja na vodni gladini. Sledijo razlogi, zakaj lahko nekateri pajki in žuželke stojijo na vodi, ter opis njihovih tehnik hoje po vodi. Glavnina članka se osredotoči na velike hodce po vodi s poudarkom na hidrodinamskem modelu, ki se uporablja za opis hoje kuščarja baziliska (lat. *Basiliscus basiliscus*) po vodi. Na koncu je podana razlaga, zakaj človek ne more hoditi po vodi, predstavljenih pa je tudi nekaj rešitev, ki bi mu to morda lahko omogočile.

1. Introduction

Animals usually move in four main places: in water, on and below solid surfaces and in air. Some animals are able to successfully combine the motion in two or more of these places, however, only a few animal species exist which can also control their motion on the surface of water. The instantaneous hydrodynamic force acting on a water-walking creature can be described in terms of forces related to form drag, buoyancy, accelerated water mass, viscosity, water surface curvature and Marangoni effect. The last three are small scale forces and are only used by arthropods to propel themselves on the water surface. Some spiders and insects, for example, exploit their hydrophobic hairy coats to get enough curvature force. Furthermore, some beetles can release chemicals behind them to locally reduce surface tension, which propels them forward by exploiting the Marangoni effect. Large water-walkers, on the other hand, rely on form drag, buoyancy and accelerated water mass. Their typical example are water-walking birds, who usually run across water by slapping the surface with their feet, with the additional assistance of their wings. Among all water-walkers, basilisk lizard is the one whose anatomy is the closest to the human's. Studying the ability of some animals to walk on water with special attention given to the hydrodynamic model for a basilisk lizard thus allows us to discuss the human ability to walk on water.

2. Introduction to physics of walking on water

The instantaneous force \mathbf{F} acting on an object striking a free surface of a fluid with surface tension σ is composed of an area contribution and an edge contribution, giving

$$\mathbf{F} = \int_S \mathbf{T} \cdot d\mathbf{S} + \int_C \sigma d\mathbf{l}, \quad (1)$$

where \mathbf{T} is the hydrodynamic stress tensor. In the approximation of incompressible fluid it can be calculated as

$$\mathbf{T} = -p\mathbf{I} + \mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^\top \right], \quad (2)$$

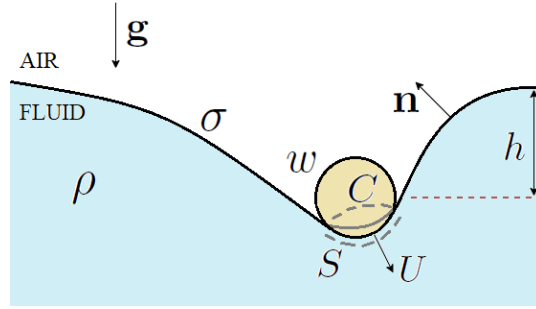


Figure 1. An illustration of the driving leg of a water-walker. An object of characteristic size w strikes the free surface with velocity U . The fluid surface’s main characteristics are surface tension σ and surface’s shape, which is described by the unit normal \mathbf{n} . Based on a sketch from [1].

where p is the fluid pressure, μ the dynamic viscosity and \mathbf{u} the fluid’s local velocity. As shown in Fig. 1, S is the area of the body in contact with the fluid and C the line separating the fluid from the body [1].

If the viscosity is negligible, the pressure is related to the velocity fields through the non-stationary Bernoulli equation

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\mathbf{u}|^2 + \frac{p}{\rho} - \mathbf{g} \cdot \mathbf{x} = \text{const.} \tag{3}$$

Here Φ is the velocity potential ($\mathbf{u} = \nabla\Phi$), ρ fluid’s density, \mathbf{g} gravity and \mathbf{x} the position of the body. Combining Eqs. (1), (2) and (3) and applying Stokes theorem¹ to re-express the second part of Eq. (1) leads to

$$\mathbf{F} = \int_S \left[\left(\rho \frac{\partial\Phi}{\partial t} + \frac{1}{2}\rho|\mathbf{u}|^2 - \rho\mathbf{g} \cdot \mathbf{x} + \sigma(\nabla \cdot \mathbf{n}) \right) \mathbf{n} - \nabla\sigma \right] dS. \tag{4}$$

Using the relation $\partial\Phi/\partial t = d\Phi/dt - (\mathbf{u} \cdot \nabla)\Phi$, we can estimate the magnitudes of terms of Eq. (4) with added viscous force as

$$|\mathbf{F}| \sim \underbrace{\rho U^2 A}_{\text{form drag}} + \underbrace{\rho g h A}_{\text{buoyancy}} + \underbrace{\rho V \frac{dU}{dt}}_{\text{virtual mass acceleration}} + \underbrace{\mu U A}_{\text{viscosity}} + \underbrace{\sigma \frac{1}{w} A}_{\text{curvature}} - \underbrace{\nabla\sigma A}_{\text{Marangoni}} \tag{5}$$

where we use the characteristic leg speed U , the characteristic area A , volume V and width w of the body, which is in contact with the fluid, and the mean leg depth h below the unperturbed fluid’s surface. All forces of Eq. (5) help² water-walkers to stay afloat and successfully move on the water surface. Water-walking animals rely on various combinations of these forces. Large water-walkers, e.g. some birds and lizards, rely on a combination of form drag, virtual mass acceleration and buoyancy. On the other hand, combinations used by small water-walkers, i.e. spiders and insects, differ almost from species to species [1].

3. Arthropods

3.1 Standing on water

Unlike large water-walkers, most arthropods are able to rest on water with no movement at all. They are usually denser than water, so they cannot rely on buoyancy to keep them on the surface. Those

¹ $\int_C \sigma d\mathbf{l} = \int_S [\sigma(\nabla \cdot \mathbf{n})\mathbf{n} - \nabla\sigma] dS$

²The minus sign at the Marangoni part just indicates that water-walkers are accelerated from the area with lower surface tension towards the area with higher surface tension. This is called Marangoni effect.

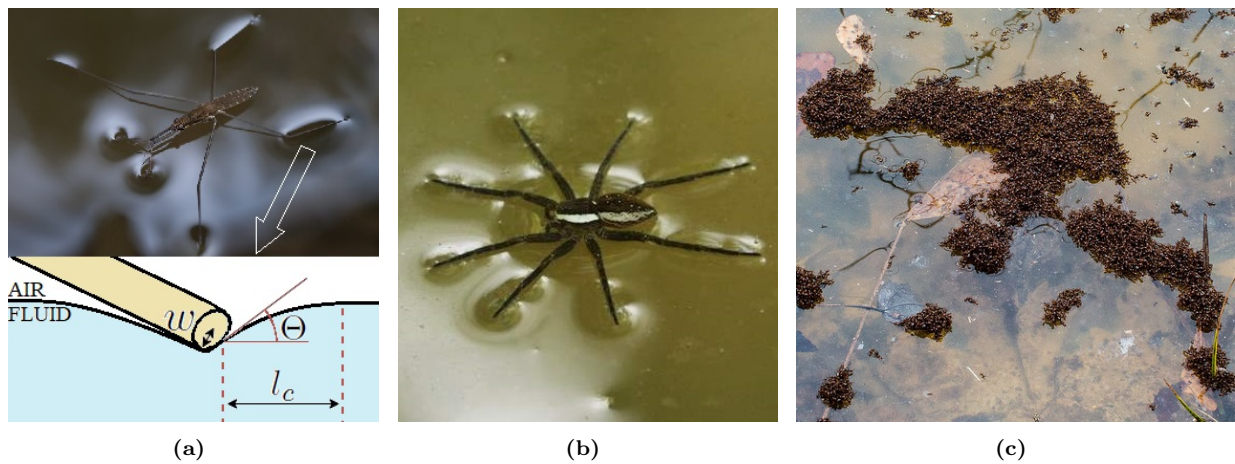


Figure 2. a) Water strider. In contact with hydrophobic material, the water surface generates a convex meniscus with maximum angle Θ and capillary length $l_c = \sqrt{\sigma/\rho g} \approx 2.6$ mm. For a long and thin body ($w \ll l_c$), e.g. the leg of a water strider, its weight is mostly supported by the curvature force per unit length, $2\sigma \sin \Theta$. b) Fisher spider. While the water strider gets enough support through its legs, the fisher spider keeps its trunk in touch with the water surface as well. c) Ant rafts. Single ants cannot survive floods, so they stick together in larger groups. Photographies adapted from [8–10]. Sketch based on a sketch from [1].

arthropods, who can survive floating on water on their own, have mostly evolved water-repellent coats, so they use water's surface tension to stay above the surface as described in the caption to Fig. 2a. The magnitude of water's surface tension is approximately $70 \mu\text{N}/\text{mm}$, which is negligible for large water-walkers, but it is sufficient for arthropods (e.g. a mosquito weighs about $1 \mu\text{N}$) [1, 2].

We should bear in mind that the sketch in Fig. 2a is actually a major simplification. In reality, a body in direct contact with water surface has to be much thinner than the water's capillary length. For this purpose, insects and spiders have their legs covered with small hairs. For example, the water strider leg touches the water surface for about 0.3 cm, its leg hairs have a length of the order of $100 \mu\text{m}$ and width of order $1 \mu\text{m}$. Hundreds of these hairs penetrate the water surface, so there is enough curvature for the entire trunk to stay above it [2].

It is worth mentioning that not all arthropods which manage to stay above the surface exploit surface tension. There are some species of ants that are only able to survive there as a collective. During floods, individual ants are doomed, however if the entire colony link their feet together, they form a water-repellent raft, like the one shown in Fig. 2c. The ant raft relies on buoyancy, since its average density is about 15% of water's. The colony can survive for months in such formation [2].

3.2 Moving on water

There are more than a million insect species, of which about 90% are able to fly and 3% are able to swim. Only about 1200 species, however, can control their movement on water [2]. And they do it in various ways.

Some insects change their propelling methods as they age. A juvenile water strider quickly brushes the water surface with its legs, causing almost no surface deformation. On the other hand, an adult water strider, like the one in Fig. 2a, simply rows at the water surface using only its middle pair of legs. One such row lasts about 0.03 s [3].

The fisher spider in Fig. 2b knows three different ways of locomotion on water surface. It can row at the surface using its middle pairs of legs similarly to the adult water strider. For a faster movement at the surface it gallops by using all legs. The third way of movement is leaping. For a leap, the fisher spider drives its eight legs downward simultaneously, deforming the free surface to depths of about 0.3 cm. The curvature force is sufficient for the spider to leap to heights of several

body lengths. It then lands on splayed legs and its underbelly [3].

If endangered, some insects, especially beetles, release lipids in order to achieve surface tension gradients, which propel them forward. They can locally reduce the magnitude of surface tension by more than 30 %, so Marangoni effect is sufficient for them to reach relatively high speeds. For example, *Microvelia*'s peak speed, caused by Marangoni effect, is about 17 cm/s, which is twice as much as its peak walking speed [1, 2].

4. Large water-walkers

As mentioned in chapter 2., large water-walkers rely on a combination of form drag, virtual mass acceleration and buoyancy. The distribution of the amounts of these contributions differs from species to species and it in general does not depend on whether a certain large water-walker can float on water without any movement or not.

If we define walking on water as propelling oneself at the surface with the majority of its body above the water, then the largest water-walkers are dolphins. As they vigorously flap their tails back and forth, dolphins can propel themselves along the water surface with only their tails beneath it, as shown in Fig. 3b. Tail-walking, as such movement is called, has only been observed in animals with characteristic lengths of 1 – 3 m. The minimum length can be explained by a simple experiment: if we try to balance a vertically positioned rod, e.g. a pencil, on our finger, we will find that it is much easier to balance a long rod than a short one. Thus too short creatures cannot tail-walk, since they are too unstable to do that. The maximum length can be explained by the biological fact that living creatures are able to generate forces proportional to their body size squared, L^2 . To tail-walk, one must generate a force almost equal to its weight, which scales as L^3 , meaning that the force to weight ratio scales as $1/L$ [1].

Other large water-walkers walk on water in a more anthropomorphic way. Various waterfowl are able to sprint across the surface by vigorous slapping of the surface. Some birds do so to prepare for takeoff and use their wings to generate enough lift. The western grebe sprints across water without using its wings during its mating ritual, as shown in Fig. 3a. Some ducks propel themselves along the water surface by slapping with both their feet and wings. Also some species of lizards are able to run short distances over water [1]. In the following subsection, the motion of the basilisk lizard is discussed in detail, since it can be used as a good model to predict the human ability to walk on water [4].

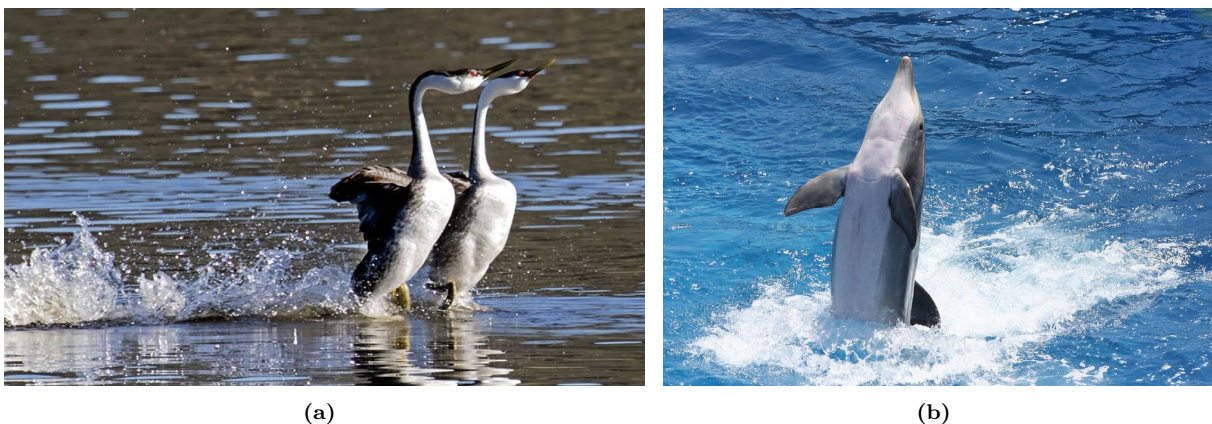


Figure 3. a) A couple of western grebes during their mating ritual. b) A tail-walking dolphin. Adapted from [11] and reproduced from [12].

4.1 Basilisk lizard

The basilisk lizard can sprint across the water surface at speeds of approximately 1.6 m/s [1]. A hydrodynamic model for the water-walking basilisk lizard consists of three phases, demonstrated in Fig. 4: slap, stroke and protraction. A step begins when the lizard swings its foot and slaps the water surface. As the lizard strokes downwards, an air cavity is produced. At the end of the stroke, the lizard pulls its foot upwards and prepares for the next step [5].

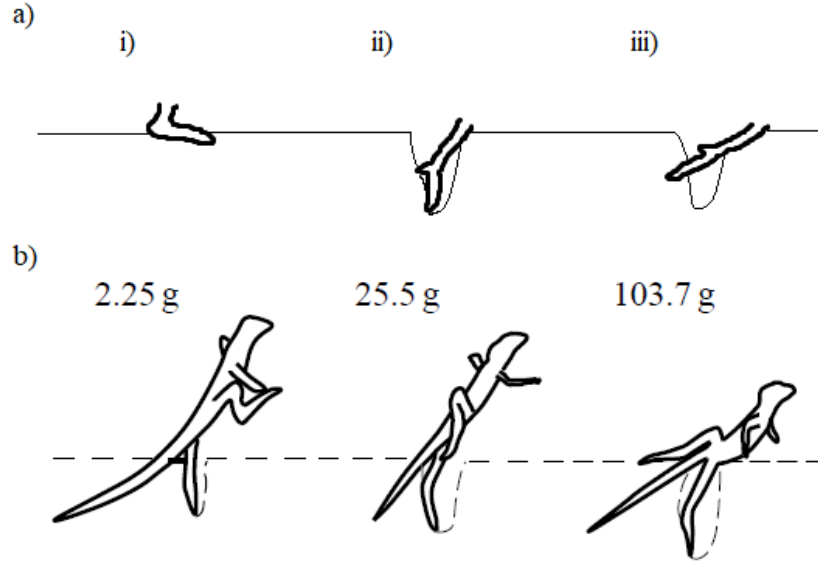


Figure 4. a) Three phases of a lizard's step. i) Slap. At the instant of impact, the lizard's foot is parallel to the water surface. ii) Stroke. After the impact, an air cavity is produced as the lizard strokes downwards and backwards. iii) Protraction. To successfully end a step, the lizard must pull its leg out of the water before the air cavity is sealed. b) Lizard's body position, when its foot reaches the lowest point within the step. As the lizard's mass grows, the relative distance between its center of mass and the water surface level decreases. Based on sketch from [5] and adapted from [6].

If we assume that a basilisk lizard only has one foot in the water at a time and that the time both its feet are out of the water is zero, then the minimum impulse for a lizard to stay above the surface is

$$J_{\text{MIN}} = Mg t_{\text{STEP}}, \quad (6)$$

where M is the lizard's mass and t_{STEP} is the duration of its step. The minimum impulse is achieved by a sum of slap impulse, J_{SLAP} , and stroke impulse, J_{STROKE} . Analogously to the virtual mass acceleration part in Eq. 5, we get

$$J_{\text{SLAP}} = m_{\text{VIRTUAL}} u_{\text{SLAP}} = \frac{4}{3} r_{\text{EFF}}^3 \rho u_{\text{SLAP}}, \quad (7)$$

where m_{VIRTUAL} is the virtual mass of water accelerated during the impact and u_{SLAP} is the impact speed of the lizard's foot, which is also the velocity to which m_{VIRTUAL} is accelerated. Here r_{EFF} is defined as the radius of a disk that would generate the same relationship between the slap impulse and impact speed as the lizard's foot [4–6]. As the foot strokes downwards, the hydrostatic pressure and the pressure due to the inertia of water create a drag force, $F(t)$. Analogously to the form drag and the buoyancy parts in Eq. 5, we have

$$F(t) = SC_D \left(\frac{1}{2} \rho u^2(t) + \rho g h(t) \right), \quad (8)$$

where S is the slap surface area, C_D is the water-entry drag coefficient, which can be approximated as $C_D \approx 0.7$ for all sizes of basilisk lizard, and $h(t)$ is the depth of the foot below the water surface.

To calculate its vertical component we multiply $F(t)$ by the cosine of the angle $\phi(t)$ between a normal to the foot and the vertical. The stroke impulse is thus calculated as

$$J_{\text{STROKE}} = \int_0^{t_{\text{STROKE}}} F(t) \cos \phi(t) dt, \quad (9)$$

where t_{STROKE} is the stroke duration. Measurements done on medium sized³ lizards showed, that the average sum of impulses was 102 % of the needed in equation (6), which confirmed that the presented model is very accurate for basilisk lizard. Slap contributes $18.3 \pm 2.72\%$ and stroke contributes $83.9 \pm 12.8\%$ of the needed impulse, which means that basilisk lizards mostly support their weight by stroke impulse [4, 5].

To successfully finish the step, t_{STEP} must be shorter than the time t_{SEAL} in which water refills the air cavity; t_{SEAL} can be calculated as

$$t_{\text{SEAL}} = \tau \sqrt{\frac{r_{\text{EFF}}}{g}}, \quad (10)$$

where τ is a dimensionless parameter depending on the shape we approximate the foot with. In our case, we approximate the lizard's foot by a disk, which gives us $\tau = 2.285$. Medium sized lizard's t_{STEP} usually ranges from about 0.05 s to 0.07 s, which is less than measured $t_{\text{SEAL}} = 0.08 - 0.09$ s. Here we should point out, that if the foot were round rather than flat, one could approximate it with a sphere corresponding to $\tau = 1.74$, which would probably lead to too short t_{SEAL} for a lizard to stay afloat [4, 5, 7]. As shown in Fig. 5, the basilisk lizard's leg and foot length grow linearly with its mass and its foot shape stays the same. However, the leg and foot growth is not exactly proportional to weight gain, so walking on water becomes more difficult for a basilisk lizard as it grows. If t_{STEP} in Eq. 6 is replaced by t_{SEAL} , then a 2 g lizard can generate a maximum upward impulse that is more than twice the value needed to support its body weight (225 %). In contrast, a 200 g lizard can just barely support its body weight (111 %) [6].

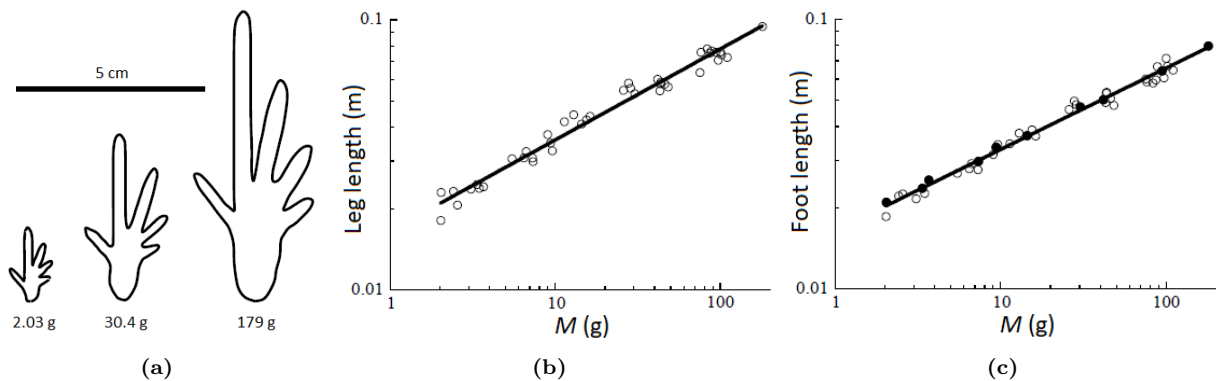


Figure 5. Development of basilisk lizard's foot shape, leg length and foot length as the lizard grows. Adapted from [6].

4.2 Human

Shortly before dawn Jesus went out to them, walking on the lake. When the disciples saw him walking on the lake, they were terrified. "It's a ghost," they said, and cried out in fear.

Matthew 14:25-26, New International Version

³Measurements were done on lizards with masses of about 90 g.

Walking on water has always been one of Man's greatest desires. People have perceived it as supernatural, as illustrated in the citation above. Man's attempts to walk on water have relied on flotation devices – one of them was even designed by Leonardo da Vinci – however, the task has never been fulfilled [1, 2].

First things first: could a human stand on water? In order to stand on water, a human should be supported by the curvature part of the Eq. (5). If we assume that $A/w \sim P$, where P is the foot perimeter, then a $M = 70$ kg person would require feet of perimeter $P = Mg/\sigma \approx 10$ km [1].

What about running on water? Typical human's r_{EFF} is about 0.1 m, which in Earth conditions gives $t_{\text{SEAL}} = 0.23$ s, while sprinters t_{STEP} is typically 0.26 s, so a typical sprinter is theoretically unable to perform a single successful step on water surface. However, if we neglect this part of calculation and continue computing, we reach a shocking result. An 80 kg person with $t_{\text{STEP}} = 0.26$ s would need to generate a minimum impulse of 204 N s. If he slaps the water surface and then strokes his feet with $r_{\text{EFF}} = 0.1$ m directly downwards to half his leg length of 1 m, he would require $u_{\text{SLAP}} = 30$ m/s to reach the minimum impulse [5]. If the person decides to reduce this speed to humanly possible 10 m/s, the surface area of his feet should be of order 1 m². However, such gigantic fins that would increase the feet area enough would not allow a human to run at such high speed in a stable way [1].

4.2.1 Reduced gravity

All the calculations presented in this article so far have proven that walking on water is impossible for a human on Earth. However the authors of [4] got the idea to try out the human ability to run on water at a gravity lower than Earth's.

To get closer to the basilisk lizard's anatomy, they slightly enlarged a human's feet by using fins shown in Fig. 6a with $r_{\text{EFF}} = 0.17$ m, so the r_{EFF} to leg length ratio was about 0.21, which is similar to basilisk lizard's 0.23. When using $M = 66$ kg, $r_{\text{EFF}} = 0.17$ m, $u_{\text{SLAP}} = 2.504$ m/s and $t_{\text{STROKE}} = 0.295$ s, the basilisk lizard model predicts that it is possible to run on water for $0 < g \leq 2.16$ m/s². Interestingly, the model predicts that 82 % of the total impulse is contributed by the stroke at $0.22 \cdot g_{\text{EARTH}}$, which is similar to the middle sized basilisk lizard in Earth conditions. The model also shows that the maximum body mass compatible with running on water at the gravity of the Moon, which is $0.16 \cdot g_{\text{EARTH}}$, with $t_{\text{STROKE}} = 0.29$ s is 73 kg [4].



Figure 6. a) The fins used in the experiment of humans running in place at reduced gravity. b) The actual footage of the experiment. Adapted from [4].

In the experiment, participants were wearing the previously mentioned fins while being attached to a constant force unloading system above a small pool where they tried to run in place as shown in Fig. 6b. Within the experiment six persons experienced different levels of simulated gravity

(10 – 25 %). The results of the experiment are demonstrated in Fig. 7. It proved the theoretical result that the maximum gravity for a human with fins to run on water is about $0.22 \cdot g_{\text{EARTH}}$. The fact that not all subjects were able to run at this or slightly lower gravity may be explained by the actual experimental conditions: the experiment was done on Earth, so the only ones experiencing lower gravity were humans, while water was still influenced by Earth's gravity and thus had a shorter t_{SEAL} than it would have at a lower gravity [4].

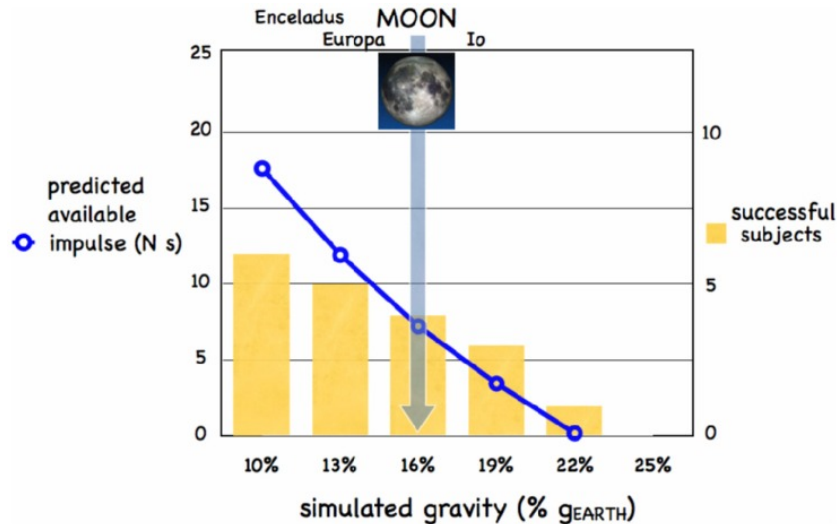


Figure 7. Results of the described experiment. The blue line represents theoretically predicted available impulse, calculated as $J_{\text{SLAP}} + J_{\text{STROKE}} - J_{\text{MIN}}$. The bars represent the number of subjects, out of six that were able to avoid sinking at different simulated gravity values. Reproduced from [4].

5. Conclusion

As shown above, all models used to explain the ways water-walkers survive on water surface originate from only one equation, whose terms are of different importance for different animals. The ways animals walk on water are highly diverse: from gently rowing water striders to compressing and leaping spiders and violently slapping birds and lizards. Basilisk lizard's way of walking on water is well understood and explained by a straightforward model. The ability of the hydrodynamic model for the basilisk lizards to be successfully applied to humans, who are about 10-times larger and 500-times heavier than them, is astonishing.

According to the experiment, humans should be able to walk on water on Moon, so one day walking on water might become an everyday thing for people living there.

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Walking on Water

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