

QUANTUM DECOHERENCE

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In this article the the process of environment-induced quantum decoherence will be studied from the perspective of quantum-to-classical transition. The formalism of density operators will be briefly introduced. Methods for finding states that are minimally influenced by the interaction with environment will be mentioned and loss of certain quantum properties in open systems explained. Some real-world experiments and examples will be given, among which the commonly encountered example of decoherence due to scattering will be described in more detail.

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Članek bo preučeval pojav kvantne dekoherence s perspektive prehoda iz kvantne v klasično mehaniko. Vpeljal bo formalizem gostotnih operatorjev. Omenil bo metode za iskanje kvantnih stanj, na katere okolje minimalno vpliva. Pojasnil bo izgubo nekaterih kvantnih lastnosti odprtih sistemov. Navedel bo nekaj zgledov in poskusov iz resničnega sveta, med katerimi bo največ pozornosti posvetil dekoherenci zaradi sipanja.

1. Introduction

While the quantum theory continues to give exceptionally accurate results, some of its aspects remain elusive from its very beginnings. When the superposition principle was first introduced a hundred years ago, it appeared to give completely counter-intuitive or even ridiculous predictions that seemed impossible to unify with apparently classical framework. Yet so far every seemingly paradoxical prediction of quantum theory has turned out to hold true and it appears that our intuition about what nature ought to be was mistaken. What makes our everyday world appear so classical then? Why do we not observe macroscopic non-local phenomena, interference patterns with everyday objects or superpositions of different types of fruits? After all, quantum mechanics does not fundamentally forbid it.

In attempt to answer those questions we have to consider interaction between a system and it's environment. In this article we will explore the phenomenon of *environment-induced quantum decoherence* (hereinafter referred to as only quantum decoherence), which essentially means the loss of "quantum behavior" due to the interaction with environment. The first to explore the theory of decoherence was a German physicist H. Dieter Zeh in 1970. Although not very widespread at first, the field has been subjected to extensive research in the last decades, mostly due to its application in the science of quantum computation. We will tackle the problem of quantum-to-classical transition and quantum measurement and see to what extent they can be solved with our current knowledge.

2. The Formalism of Composite Systems and the Density Operator

In the theory of decoherence we will study the effects of the inevitable interaction of a known quantum system with its environment. We will therefore study the quantum system containing both the quantum system of interest and the environment. In this chapter we introduce the mathematical formalism that is well suited for treatment of composite quantum systems and the statistical properties of its subsystems. Note that in this chapter we will only deal with finite-dimensional Hilbert spaces, but for our uses a similar formalism holds for infinite-dimensional spaces.

2.1 Composite Systems

Suppose we have two Hilbert spaces \mathcal{A} and \mathcal{B} with dimensions m and n respectively. Let $\{|a_i\rangle\}$ be an orthonormal basis for \mathcal{A} and $\{|b_j\rangle\}$ an orthonormal basis for \mathcal{B} . We define their tensor product $\mathcal{A} \otimes \mathcal{B}$ as a Hilbert space of dimension mn with the set of its orthonormal basis vectors written as

$$\{|a_i\rangle \otimes |b_j\rangle \mid i = 1, 2, \dots, m; j = 1, 2, \dots, n\}. \tag{1}$$

Here we say that $|a_i\rangle \otimes |b_j\rangle$ is a tensor product of vectors $|a_i\rangle$ and $|b_j\rangle$. Tensor product is linear in both its factors

$$|\psi\rangle \otimes (\alpha_1 |\phi_1\rangle + \alpha_2 |\phi_2\rangle) = \alpha_1 |\psi\rangle \otimes |\phi_1\rangle + \alpha_2 |\psi\rangle \otimes |\phi_2\rangle \tag{2}$$

$$(\alpha_1 |\psi_1\rangle + \alpha_2 |\psi_2\rangle) \otimes |\phi\rangle = \alpha_1 |\psi_1\rangle \otimes |\phi\rangle + \alpha_2 |\psi_2\rangle \otimes |\phi\rangle. \tag{3}$$

For a product vector $|\psi\rangle \otimes |\phi\rangle$, the corresponding bra is

$$\langle\psi| \otimes \langle\phi| = (|\psi\rangle \otimes |\phi\rangle)^\dagger. \tag{4}$$

If the vector $|v\rangle$ can be written as $|u\rangle \otimes |v\rangle$ where $|u\rangle \in \mathcal{A}$ and $|v\rangle \in \mathcal{B}$, it is called a *product state*, otherwise it is called an *entangled state* (an example would be $|u_1\rangle \otimes |v_1\rangle + |u_2\rangle \otimes |v_2\rangle$ for $|u_1\rangle \neq |u_2\rangle$ and $|v_1\rangle \neq |v_2\rangle$). When the quantum system is in a product state, it is possible to treat both systems individually without any loss. Contrarily, if the system is in an entangled state, we cannot treat subsystems independently due to the correlations that arise between them. In this case the results of measurements performed on individual subsystems separately will not be independent. Entanglement is a completely quantum phenomenon with no classical analogue and is of crucial importance in the theory of quantum decoherence.

If A is an operator on the elements belonging to space \mathcal{A} , and B on elements belonging to \mathcal{B} , we can construct tensor product of operators A and B as follows:

$$(A \otimes B)(|\psi\rangle \otimes |\phi\rangle) = (A|\psi\rangle) \otimes (B|\phi\rangle). \tag{5}$$

The parenthesis can be safely omitted on the right hand side of the equation.

2.2 Density Operator

Instead of describing a quantum system with a state ket, we can attribute an operator to the state called the *density operator*¹. The density operator formalism is indispensable when treating subsystems of a composite system or a system we have insufficient knowledge of.

Firstly we must define the terms *pure state* and *mixed state*. A pure state is a state completely described by a state vector from the corresponding Hilbert space. By knowing what pure state the system is in, we possess maximal possible information about the system.

A mixed state is a term for describing a system which is in one of several different pure states, in each with a certain probability. It is very important to note that in such case the system is still physically in a pure state, it is merely us who are ignorant about which pure state the system is in. Suppose we have a set of n states $\{|\psi_i\rangle \mid i = 1, 2, \dots, n\}$ we know the probability p_i for the physical system to be in the state $|\psi_i\rangle$. We define the density operator ρ as a weighted sum of projectors $|\psi_i\rangle\langle\psi_i|$

$$\rho = \sum_{i=1}^n p_i |\psi_i\rangle\langle\psi_i|, \tag{6}$$

¹Commonly referred to as the *density matrix*

with the conditions

$$\sum_{i=1}^n p_i = 1; \quad p_i \geq 0. \tag{7}$$

The density operator represents a pure state only when the sum 6 has only one term. There is an important distinction between the sum over the basis vectors as $\sum c_i |\psi_i\rangle$ and the equation 6. The former represents a quantum superposition where all the states coexist, whereas the latter takes into account our ignorance about the state of the system only as a classical probability. The most crucial difference in behavior of quantum superposition and classical probability is perhaps that interference phenomena can be observed in the former.

To illustrate the difference on the density operator, suppose we have a quantum system described by a 2-dimensional Hilbert space (commonly referred to as qubit) with its orthonormal basis vectors $|0\rangle$ and $|1\rangle$. Let us define $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Let us construct two different density operators ρ_1 and ρ_2

$$\rho_1 = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) \quad \rho_2 = |\psi\rangle\langle\psi| = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1| + |0\rangle\langle 1| + |1\rangle\langle 0|). \tag{8}$$

Density operator ρ_1 represents a mixed state, as it is a weighted sum of two projectors, while the density operator ρ_2 represents a pure state. We see that both operators have the terms $|0\rangle\langle 0|$ and $|1\rangle\langle 1|$ in common. Should we write the density operator in the matrix form in the basis $\{|0\rangle, |1\rangle\}$ we would see those terms as diagonal terms. The mixed terms ($|0\rangle\langle 1|$ and $|1\rangle\langle 0|$) of the pure state are called the *interference terms* in the basis $\{|0\rangle, |1\rangle\}$. They would correspond to the off-diagonal elements in the matrix form of the density operator. Note that the interference terms are basis dependent, which means that mixed states can certainly have off-diagonal terms. It can be shown that a density operator ρ represents a pure state exactly when $\text{tr}(\rho^2) = 1$. If this criterion does not hold, the density operator represents a mixed state. Trace is a basis independent operation, therefore the choice of basis does not affect whether the density operator corresponds to a pure or a mixed state.

While the density matrix is a very useful way to describe a quantum system, it does not provide us with the exact set of possible pure states. There is a lot of different density operator decompositions which give us the set of possible pure states with corresponding probabilities. While we may not know from the density operator alone what is the set of possible pure states, there is no measurable property that would allow us to differentiate between different possible sets.

2.3 Partial Trace

Suppose we have a state described by a density operator $\rho_{\mathcal{AB}}$ on a composite Hilbert space $\mathcal{A} \otimes \mathcal{B}$ but the subsystem corresponding to \mathcal{B} is somehow inaccessible, therefore we are only interested in the properties of the subsystem corresponding to \mathcal{A} . In other words, we only want to use operators of the form $S \otimes \mathbb{1}$. If our state was a simple pure product state there would be no difficulty in acquiring all the measurable statistics of the chosen subsystem. However, if the system is in an entangled state, we use an operation called *partial tracing*. This is an operation on the density operator similar to tracing, except that it applies only to a chosen subsystem. When we perform a partial trace on operator $\rho_{\mathcal{AB}}$ over \mathcal{B} we get a new operator $\rho_{\mathcal{A}}$ which contains all the statistics we could possibly acquire with performing measurements on system \mathcal{A} , provided we do not have any information of any measurements on system \mathcal{B} . We define partial trace over \mathcal{B} on the operator $|\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi|$ as

$$\text{tr}_{\mathcal{B}}(|\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi|) = |\psi\rangle\langle\psi| \cdot \sum_{j=1}^n \langle b_j | \phi \rangle \langle \phi | b_j \rangle, \tag{9}$$

where $|b_j\rangle$ are the basis vectors of \mathcal{B} . Along with equation 9 we require that partial trace is a linear operation. We are now equipped for computing a partial trace on a general density operator. A density operator acquired with performing a partial trace on a larger density operator is called a *reduced density operator*.

2.4 Proper and Improper Mixtures

In chapter 2.2 we introduced the concept of mixed states. We were talking about states with unknown exact quantum state. Such statistical mixtures are called *proper mixtures*. Suppose we have a density operator of a pure state on composite system and we trace out one of the subsystems. We can end up with a density operator that behaves like one of a mixed state. We say that the remaining subsystem is an *improper mixture*, since it behaves like a mixed state, even though the system is physically in a pure state.

3. The Problem of Quantum-to-Classical Transition

3.1 The Problem

Ever since the formulation of quantum theory its non-intuitive nature has boggled the minds of physicists and philosophers alike. The superposition principle works indisputably well with analysis of quantum systems and yet it is fundamentally difficult to grasp in everyday classical world. While being completely used to interference phenomena with, say, photons, interference experiments with macroscopic objects do not seem possible.

An even more mind-boggling problem than the ones mentioned above is the problem of the wave function collapse. In the Copenhagen interpretation there exists a so-called *Heisenberg cut* that separates the quantum system and the classical measuring device in experiments. The Heisenberg cut seems arbitrary to some degree and is certainly unsettling, since we can consider both the measured system and the measuring device as one larger quantum system that evolves unitarily. It seems natural then to consider the whole universe as an isolated quantum system that evolves unitarily, however, difficulties soon arise when we consider that the experiments we perform (as part of this unitarily evolving universe) still happen to have definite outcomes. This is commonly called the *measurement problem* and is the focal point of different interpretations of quantum mechanics.

3.2 The Schrödinger's Cat

The problem is popularly illustrated with the paradox of Schrödinger's cat. The scenario is as follows: there is a cat in a completely sealed and opaque box, along with a radioactive source, radioactivity monitor, hammer and a flask of strong poison. If the radioactivity monitor detects an emitted particle from the radioactive source (a process which is completely random and is in no way predetermined), it triggers the hammer so that it smashes the flask, releasing the poison into the air and instantly killing the cat. In a simplification of the problem we have a vector from the Hilbert space with the basis $\{|alive\rangle, |dead\rangle\}$ corresponding to the cat as whole. It seems very natural that the cat will be *either* dead *or* alive, but quantum theory does not forbid states such as $(|alive\rangle + |dead\rangle)/\sqrt{2}$. The main question that arises is why it is so natural that the cat will be found in either $|dead\rangle$ or $|alive\rangle$ state and not in either $(|alive\rangle + |dead\rangle)/\sqrt{2}$ or $(|alive\rangle - |dead\rangle)/\sqrt{2}$ for example. What is the process that eliminates the latter states and makes them nonobservable on macroscopic scales? Does Schrödinger's cat collapse into either $|alive\rangle$ or $|dead\rangle$ when we open the box, or was it in one of those states before we even touched the box? If it does collapse in such manner, what makes it collapse into the $\{|alive\rangle, |dead\rangle\}$ basis?

3.3 The Three Subproblems

We can decompose the problem of quantum-to-classical transition into three parts, following Schlosshauer [1]:

- *The problem of the preferred basis* – Why do we usually observe objects with well defined position rather than superposition of positions? Why do we observe cats only as dead or alive and not a superposition?
- *The problem of nonobservability of interference* – Why can we not observe interference phenomena with macroscopic objects?
- *The problem of outcomes* – If the time evolution of the universe is unitary, then why do we observe definite outcomes of experiments? Why do we not observe superpositions of different measurements? Is there truly a wave function collapse and if so, how exactly does it happen?

The theory of decoherence addresses the problem of the preferred basis and the problem of nonobservability of interference. The problem of outcomes remains mostly a matter of interpretations.

4. Quantum Decoherence

In this chapter we will deal with the loss of interference terms – a process called *quantum decoherence*. Throughout our analysis we will assume that system always evolves unitarily according to Schrödinger's equation.

Suppose we have a quantum system, that can be isolated at first. If we are to measure any property of that system we will inevitably have to make it interact with the environment – the measuring device. This is a crucial difference between a classical and a quantum measurement. In the classical world we can usually measure quantities virtually without disturbing the observed system. In the quantum world, interaction between the observed system and the measuring device is inevitable and irreversibly affects the state of the composite system. The measuring device and the observed system become *entangled*. Much like above, an open quantum system gets entangled with its environment (like a dust particle that gets entangled with the light or air molecules scattering off it), which results in our inability to fully describe the quantum system with a pure state ket, no matter how weak the interaction was (which is completely unlike in classical mechanics, where there is no entanglement and we can effectively ignore very weak interactions). We could say that environment is effectively a measurement device of some sort.

4.1 Von Neumann Measurement Scheme

Von Neumann developed an ideal measurement scheme where he treated both the quantum system and the measuring device as quantum objects. As we shall see, it has direct relation to the theory of quantum decoherence.

Suppose we have the measured quantum system described by a Hilbert space \mathcal{S} with basis vectors $\{|s_i\rangle\}$ and a measuring device described by a Hilbert space \mathcal{M} with basis vectors $\{|m_i\rangle\}$. Let us assume the measuring device is initially in a ready state $|m_r\rangle$. Suppose the measuring device has a pointer that moves to position i if the state $|s_i\rangle$ is measured to be in state $|s_i\rangle$. In that case the measuring device would be in state $|m_i\rangle$. Summarizing the process of measurement we have

$$|s_i\rangle |m_r\rangle \longrightarrow |s_i\rangle |m_i\rangle . \quad (10)$$

We see that we end up with a separable product state. Suppose we start with a general $|\psi\rangle$ describing the measured system

$$|\psi\rangle |m_r\rangle = \left(\sum_i |s_i\rangle \right) |m_r\rangle = \sum_i |s_i\rangle |m_i\rangle. \quad (11)$$

Here we see that the final state of the system is an entangled state. This kind of measurement is called ideal or non-demolition measurement because it does not disturb the state of the measured system.

4.2 Environment as a Measuring Device

Let us consider an object, say a physics textbook, laying on a table. Every second a colossal amount of air molecules and photons scatter off the textbook. A different orientation of the textbook would without doubt affect the final state of air molecules and photons. At least to some degree we will be capable of deducing the book's orientation from the scattered particles alone (after all, that's how we use our vision). We can say that environment carries away information about the state of the textbook as a consequence of quantum entanglement.

Let us look at a more general case and suppose we have a quantum system $|\psi\rangle$ described by a superposition of two orthogonal quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$, so that $|\psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$. Let $|E_0\rangle$ describe the environment before its interaction with the quantum system and let $|\psi_1\rangle$ and $|\psi_2\rangle$ interact with the environment in a way completely analogous to von Neumann measurement scheme

$$|\psi_1\rangle |E_0\rangle \longrightarrow |\psi_1\rangle |E_1\rangle \quad (12)$$

$$|\psi_2\rangle |E_0\rangle \longrightarrow |\psi_2\rangle |E_2\rangle. \quad (13)$$

If we let $|\psi\rangle$ interact with the environment we have

$$|\psi\rangle |E_0\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) |E_0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|\psi_1\rangle |E_1\rangle + |\psi_2\rangle |E_2\rangle). \quad (14)$$

We are, of course, mostly interested in the behavior of the chosen quantum system rather than the composite system. If we write the density operator of the final state and trace out the environmental part we get

$$\rho_\psi = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2| \langle E_2|E_1\rangle + |\psi_2\rangle\langle\psi_1| \langle E_1|E_2\rangle). \quad (15)$$

Suppose $|\psi\rangle$ represents a textbook in two different orientations and $|E_0\rangle$ represents the surrounding photons and air molecules. Provided we have enough photons of sufficiently small wavelength we will certainly be able to visually tell apart different orientations of the textbook. That means states $|E_1\rangle$ and $|E_2\rangle$ will be almost perfectly distinguishable, which means that $\langle E_1|E_2\rangle \approx 0$ (the approximation is very good due to the usually extremely large number of degrees of freedom involved). It follows that the interference terms in equation 15 vanish, leaving us only with an improper mixture of $|\psi_1\rangle$ and $|\psi_2\rangle$. This is the very essence of decoherence.

We worked with such states $|\psi_1\rangle$ and $|\psi_2\rangle$ that behave according to equations 12 and 13. These states do not get entangled with the environment (the reduced density operator corresponding to $|\psi_1\rangle |E_1\rangle$ is the same as the density operator corresponding to $|\psi_1\rangle$ only). States $(|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$ and $(|\psi_1\rangle - |\psi_2\rangle)/\sqrt{2}$ do get entangled with the environment and are subjected to decoherence, while their sum and difference, which are again $|\psi_1\rangle$ and $|\psi_2\rangle$ respectively, do not. We wish to find states that are robust to the interaction with the environment like $|\psi_1\rangle$ and $|\psi_2\rangle$ are. Those states will not be subjected to decoherence in any way, while their superpositions will usually be quickly (usually exponentially) reduced into improper mixtures. States that satisfy such stability criterion are commonly called *pointer states*.

4.3 Environment-Induced Superselection

The process of environment “choosing” the pointer states over their superpositions is called *environment-induced superselection*, or commonly *einselection*. Our goal is to study the properties of such states and thus addressing the preferred basis problem.

Let us suppose our system $\mathcal{S} \otimes \mathcal{E}$ is composed of the observed quantum system \mathcal{S} and the much larger environment \mathcal{E} . We wish to find the pointer states $|\psi_i\rangle$ that are stable when time evolution is governed by the Hamiltonian H . In mathematical language this means that

$$e^{-iHt} |\psi_i\rangle |E_0\rangle = |\psi_i\rangle |E_i(t)\rangle, \tag{16}$$

assuming the Hamiltonian is time independent. First lets consider the case where H has the structure

$$H = S \otimes E. \tag{17}$$

If $|\psi\rangle$ is the eigenstate of operator S (and in turn of the operator e^{-iSt}) we get

$$e^{-itS \otimes E} |\psi_i\rangle |E_0\rangle = \lambda_i |\psi_i\rangle e^{-iEt} |E_0\rangle = |\psi_i\rangle |E_i(t)\rangle, \tag{18}$$

where $|E_i(t)\rangle = \lambda_i e^{-iEt} |E_0\rangle$ (E being an operator). We see that $|\psi_i\rangle$ being an eigenstate of S exactly fulfils the condition for $|\psi_i\rangle$ being a pointer state. It follows that the observables, corresponding to Hermitian operators that commute with S , remain well determined through time.

We can always decompose the Hamiltonian of the composite system into an interaction Hamiltonian H_{int} , environment self-Hamiltonian $H_{\mathcal{E}}$ and observed system self-Hamiltonian $H_{\mathcal{S}}$

$$H = H_{int} + H_{\mathcal{E}} + H_{\mathcal{S}}. \tag{19}$$

If the $H_{\mathcal{E}}$ and $H_{\mathcal{S}}$ are negligible then H_{int} dominates the time evolution of the system. In a significant number of cases, H_{int} is of the form of equation 17, where S commutes with the position operator due to the nature of inter-particle potential. Eigenstates of S will therefore be position eigenstates. The pointer states in this case will not be position eigenstates exactly because of inevitable $H_{\mathcal{S}}$ that usually has de-localizing effect, but due to H_{int} being predominant, the pointer states will be well localized. Even though we only considered a limit case, good localization of pointer states definitely seems like a step in the right direction.

In general, where neither $H_{\mathcal{E}}$ nor $H_{\mathcal{S}}$ are negligible, we could attempt to find its corresponding eigenbasis. This procedure usually happens to be very complicated or impossible in practice, so several general methods have been development in the last decades. In the scope of this article we will only mention and conceptually describe them.

Usually we are satisfied with finding very robust states over of the ones that are completely stable. With a method developed by W. Zurek called *the predictability sieve* method we introduce a measure of decoherence introduced into the system. This can be for example von Neumann entropy or density operator purity among others. We then compute the time dependency of such a measure for a large set of initial states. The states that show the least amount of decoherence introduced into the system will be the most robust. The main flaw of this method is that different measures of decoherence will choose different robust states. It has been shown that different appropriate choices of measure affect the final states minimally, assuring the method’s stability.

If we return to the textbook on a desk from chapter 4.2 and remember that we, humans, can reliably determine its orientation solely by the photons scattered off the book and absorbed by our eyes, we can reflect on the fact that we do not need to “measure” the book to know its orientation – the environment does it for us. It carries away useful information about the book – this is the information that environment “effectively measures”. It turns out that the states which enable the

environment “to measure” the largest amount of information about them are also the most robust ones. Studying the information that can be stored robustly in the environment (in a way that would make all observers agree upon what state the subsystem is in with making deductions only from the environment) has proved to be an useful tool in determining the pointer states.

5. Decoherence Due to Scattering

Scattering is along with thermal radiation the predominant source of quantum decoherence. While the number of air molecules and visible photons can be relatively easily minimized, we are still left with cosmic particles, microwave background and thermal radiation, which makes decoherence practically inevitable. We emphasize again that, unlike in the classical case, scattering affects the quantum object’s state significantly even if the effect on its center of mass is negligible. In this chapter we will summarize the results of theoretical predictions about quantum decoherence due to scattering.

5.1 Coherence and Decoherence in Position Space

Since we will now work with wave functions from continuously infinite-dimensional space corresponding to physical position we need to introduce the appropriate density operator. For a pure state $|\psi\rangle$ the corresponding density operator is $|\psi\rangle\langle\psi|$. The “matrix element” in position basis would be

$$\langle x|\rho|y\rangle = \langle x|\psi\rangle \langle\psi|y\rangle = \psi(x)\psi^*(y), \quad (20)$$

where $\psi(x)$ denotes the usual wave function in the position space. $\langle x|\rho|x\rangle$ of course corresponds to the probability density and similarly to the discrete matrix form, non-zero values far from diagonal $x = y$ represent interference. Density operators that are very spread out in the direction perpendicular to the diagonal ($x=-y$) will show great coherence in contrast to operators which are focused around the diagonal.

5.2 Characteristic Times of Decoherence in Scattering Environment

In a very detailed analysis found in Schlosshauer [1] (Chapter 3), an explicit time evolution of a wave packet in a general scattering environment is obtained. In this article we will only use the main results as an illustration of decoherence in action. In the derivation the following assumptions were made:

- There are no initial correlations between the system and the environment.
- The problem is invariant under translations of composite system.
- Center of mass of the object is not disturbed by scattering (the object of interest is much more massive than its surrounding particles)
- The rate of scattering is much higher than the characteristic rate of change of the state of the system induced by the system’s self Hamiltonian.
- The distribution of incoming particles is isotropic.

Time evolution of the reduced density operator ρ_p for the particle is obtained

$$\langle x|\rho_p(t)|y\rangle = \langle x|\rho_p(0)|y\rangle e^{-t/\tau}, \quad (21)$$

$$\tau = \tau(|x - y|). \quad (22)$$

Environment	Dust grain	Large molecule
Cosmic background radiation	1 s	10^{24} s
Photons at room temperature	10^{-18} s	10^6 s
Best laboratory vacuum	10^{-14} s	10^{-2} s
Air at normal pressure	10^{-31} s	10^{-19} s

Table 1. Theoretical estimates of decoherence timescales for a dust grain of size 10^{-5} m and a large molecule of size 10^{-8} m. Size of the particle is used as the parameter $|x - y|$ from equation 21 [1]

The characteristic decoherence time τ is a function of distance from the diagonal $|x - y|$. In general it will hold that for greater $|x - y|$ the characteristic time τ will be smaller. This will lead to the density operator becoming narrower in the direction perpendicular to the diagonal, illustrating decoherence.

Table 1 shows the theoretically predicted characteristic timescales of decoherence for differently sized particles in different environments. The size of particles is used as the parameter $|x - y|$, which is also the only property of the particles used in this derivation. Timescales of decoherence for macroscopic and mesoscopic objects are outstandingly small, much smaller than any measurable physical process in any conceivable circumstances (like dissipation). We can clearly observe that decoherence process is extremely efficient in localizing any objects that may find themselves in a scattering environment.

6. Examples and Experiments

Here we will mention a few experiments supporting decoherence theory and some examples of decoherence in action.

- First experimental verification of decoherence theory was done in 1996 ([11]) where gradual well-controlled decoherence of superposition of two energy eigenstates of $^9\text{Be}^+$ ions was observed.
- An important milestone was reached when interference was observed in experiments with large C70 molecules – in an experiment similar to the double-slit setup ([12]), it was observed that the visibility of the interference pattern decreases exponentially with pressure of surrounding gas, agreeing with theoretical predictions. The order of magnitude of pressures was 10^{-6} mbar (which is considered a good vacuum), where visibility of the interference pattern was well below 50%. This experiment directly confirmed localization due to environment induced decoherence and neatly illustrated its dramatic effects even in a good vacuum.
- In quantum computing decoherence presents a major setback in physical realization of quantum computers. That is because the advantages of quantum computing lie in using coherent superpositions that are easily destroyed even by a minimal interaction with the environment. There is a lot of active research being conducted about how to overcome this difficulty. Without going into much detail we only acknowledge that decoherence has been studied extensively in different implementations of quantum computers, for example with superconductive quantum interference devices (SQUIDs).
- An intriguing example of decoherence in action is observed in certain chiral sugar molecules, which are always observed in chirality (handedness) eigenstates, which are in fact superpositions of different energy eigenstates (corresponding to different energy) [13]. Should we try to

maintain molecules in energy eigenstates, we would find that they decohere very quickly into chirality eigenstates.

- So far we have most often come to a conclusion that decoherence usually localizes objects in position space. That, however, is not always the case. In Brownian motion it happens that particles localize in neither position nor momentum space but somewhere else in phase space. It is a prime example of neither the self Hamiltonian nor the interaction Hamiltonian being negligible.
- In nature we never observe superpositions of differently charged particles (for example a proton and a neutron). Initially this was simply postulated as a superselection rule but was subsequently explained ([9]) as a consequence of the charge's interaction with its Coulomb field, which plays the role of the environment. Charge superposition is thus quickly decohered into an improper mixture.

7. Conclusion

The decoherence theory addresses the problem of interference and the problem of preferred basis relatively successfully. The quantum-to-classical transition mystery remains largely unsolved due to the problem of collapse but einselection certainly provides us with at least some explanation. It is very tempting to think that decoherence resolves quantum superpositions into classical probabilities, but we must remember that the “classical probabilistic nature” is only a consequence of tracing out the environment leaving us with *improper mixture*. Thus we can explain the lack of interference but the system as whole remains in a pure state and doesn't intrinsically involve classical probabilities. We therefore cannot confidently say that superposition is transformed into a true “either-or” situation, we can only say it behaves as such when observing appropriately small subsystem. At this point we should emphasize that decoherence is not an interpretational or philosophical matter but an experimentally supported physical theory that, among other applications, relates to measurement problem and quantum-to-classical transition. The study of systems interacting with their environment is of course a much wider field and the term decoherence is only used to describe the loss of coherent superpositions. It is without doubt very satisfactory that we are provided with some methodology of determining pointer states in a manner that is not completely ad hoc, and that we have acquired theoretical background that explains the absence of many “nonsensical” quantum phenomena in macroscopic world.

In this article we have only scratched the surface of the decoherence theory with introducing the fundamental concepts. Many concrete models of decoherence have been developed and a number of verifying experiments have been successfully conducted. Due to its relevance in the field of quantum computing it remains an active field of research.

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