UPORABA STOLPČNIH DIAGRAMOV PRI POUKU FIZIKE

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Uporaba več različnih reprezentacij med poukom fizike lahko prispeva k boljšemu reševanju fizikalnih problemov in boljšemu razumevanju konceptov. V članku so obravnavani predvsem stolpčni diagrami in načini, kako so lahko uporabljeni v različnih fizikalnih temah.

USING BAR CHARTS IN PHYSICS TEACHING AND LEARNING

The use of multiple representations in physics learning leads to a better problem solving and understanding of physical concepts. This article is focused on bar charts and the way they can be used in different physics topics.

Kazalo

1. Introduction

Some researchers have focused on how different people solve physics problems. They also wanted to know if there are any tools that can help them to be more successful in solving problems. Let's see how experts (for example experienced physics teacher) are solving problems. When they read a problem, they firstly try to visualize and describe a given situation. They identify physical quantities that are given in a problem statement and also the ones that need to be determined. Next, they make some kind of a sketch. It is important to realize, when experts meet a completely new problem, they almost never start solving it by constructing a mathematical description of the model. They start with constructing a visual representation that contains a low-detail qualitative physical description by selecting a general method and key aspects of the problem. Then they apply general principles depending on the problem. To do this step, they use various representations. At the end, they calculate or estimate the unknown quantities. Experts use many different representations in order to obtain the solution and are able to (and often do) solve the problem in more than one way [1, 3].

Students, though in a traditional way, solve problems differently. Some of them also write down physical quantities and only few spontaneously draw a labelled sketch, but the main difference with experts is that they proceed straight to writing "formulas". They do not use multiple representations and they mostly just try to remember or search for the equations with quantities that are given in a problem statement to calculate the unknowns. To some extent this is a consequence of the type of problems that can be found in most textbooks, because they encourage the approach of solving problem mentioned above. The numerical result that they get might be correct, but they often do not understand the physical concepts, which can later prevent them from solving more difficult problems [1]. Therefore, students should be though in a different way. Students who are though to mostly (or only) use mathematical representations when solving problems, are not very successful at solving problems that assess understanding of physical concepts, even if the problems are considered to be relatively simple. By contrast, students that are thought to use multiple representations are statistically more successful in solving such problems [2].

To show that multiple representations are really useful, some findings from Physics Education Research (PER) will briefly be discussed. Students who were using bar charts during lectures and for homework were asked the following question: "Did using the energy bar charts help you learn energy concepts and solve work-energy problems? Explain why they were useful or not useful." [3]

64% of all the students (N = 67) answered that bar charts helped them understand what is happening to different types of energy in the system, and they helped them to construct correct equations. 15% of the students answered that bar charts helped them understand concepts and the energy conservation better. 10% answered that the energy bar charts are useful at the beginning of learning a new topic, but later they would rather create them only in their heads. 3% of all the students did not explain why they think bar charts are useful. Only 8% of the students thought that bar charts are not a useful representation. Analysis of this research shows that energy bar charts as a visual representation play a crucial role in helping students understand energy concepts and solve related problems [3].

Then the students were asked: "Did you (or how did you) use the energy bar charts and the multiple representations of work-energy processes to solve problems while doing homework, group problems in recitation, problems in lab, and/or on exam problems?" [3] 46% of all the asked students use multiple representations at the beginning of new topic, but later less and less. One third of the students answered that they use them most of the time. 16% of the students only use multiple representations to evaluate their work or when they feel that the problem is difficult. Therefore, 95% of all asked students answered that they are using multiple representations while solving problems and only 5% of all the students never use multiple representations [3].

Another study focused on the use of another useful representation – force diagram [2]. Researchers wanted to know, if students that were taught to use multiple representations in their course, also use them on their own, when they solve problems on test. Students were given multiple-choice problems on their exam. Tasks did not specifically ask to use force diagrams and students did not get any credits for drawing them. Force diagrams, which were drawn by the students on the test paper, were coded on a 4 level scale (0 to 3), depending on their quality. 0 was coded where there was no evidence of created force diagram, 1 when force diagram included major mistakes, 2 when force diagram had smaller mistakes and 3 when force diagram was created correctly. It must be taken into account, that data points here are not students, but the solutions that were written (on test paper questions). 125 students answered to five questions and 120 students answered to seven questions. As it can be seen in Figure 1, the percentage of responses of students who solved the problem correctly (grey bar) is the highest in group 3, which shows that those students who drew a force diagram correctly, were more likely to also interpret it correctly and obtain the right result. It can also be seen, that students who drew force diagrams incorrectly, were the least likely to solve the problem correctly, even when compared to students who had no evidence of using force diagram. This might occur due to the fact, that some students may have created force diagrams in their head.

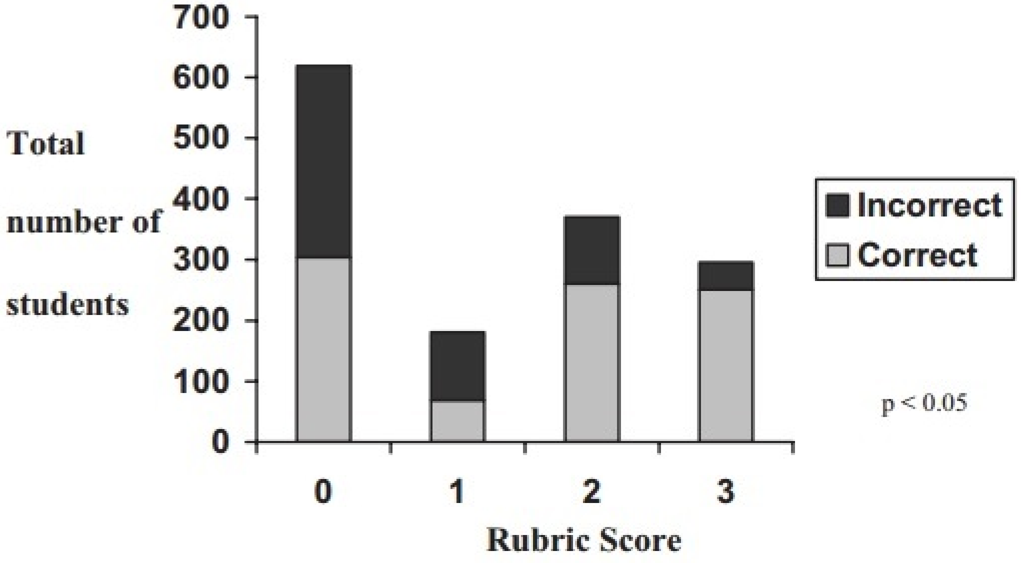

Figure 1:*The number of answers, which were put into rubrics labelled with numbers from 0 to 3* [2].

So, the researchers concluded that students who construct a correct force diagram are more likely to be successful in solving (relevant) problems than students who constructed incorrect diagram or had no evidence of using a force diagram at all [2].

When solving physics problems, different representations can be used. The following items are all regarded as representations: written representation (text), sketches, motion diagrams, graphs, field lines, ray diagrams,... and finally mathematical representation (equations), the last being the most abstract. Therefore other (picture-like) representations can be (and when teaching physics should be) used, to build bridges from concrete to more abstract representations [3].

Motion of a moving particle can be represented by a dot diagram (dots are drawn at the position of the object in equal time intervals). To show movement in a more thorough way, velocity vectors and velocity change vectors (that have the same direction as the acceleration) can be added. Such representation is called motion diagram (Figure 2a). Motion can also be represented with data tables and graphs, which are representations that are more abstract then motion diagrams and therefore a step closer to a quantitative treatment of a problem.

When solving dynamic problems, force diagrams, sometimes also called free body diagrams, can be used (Figure 2b). The object of interest is represented as a dot (assume point mass model) with drawn labelled force arrows.

Sometimes magnetic or electrical fields can be represented by using field lines. Such lines include information about the direction of the field (tangent to the line in any point of space) and also allow comparison of the magnitudes of the fields since field lines density is proportional to the magnitude (Figure 2c).

For representing the propagation of light, mostly when solving problems related to lenses and mirrors, rays (that are abstract representation for very narrow light beams) can be used. The construction that includes characteristic rays is called a ray diagram (Figure 2d).

Spacetime diagrams are special graphs that can be useful in problems related to special relativity. On such diagrams, the meaning of axes is reversed; time is on the vertical axis and the position on the horizontal axis. Therefore the slope of the line represents inverse of speed. This kind of line is called a world line (Figure 2e) [4].

A Faynman diagram is a representation that helped physicists to understand process in quantum field theory by helping visualise interaction of sub-atomic particles. Faynman diagrams (Figure 2f) give a simple visualization of what would otherwise be an arcane and abstract formula.

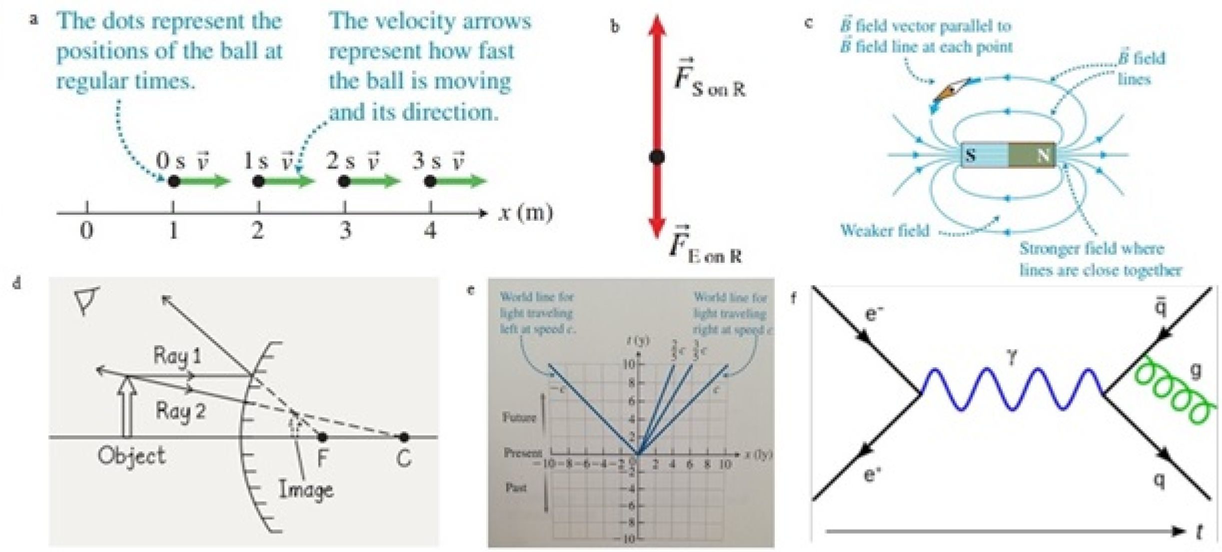

Figure 2: Different representations. (a) Motion diagram for a body that is moving with a constant speed in the same direction. (b) Force diagram for a body on which two forces are exerted. From this representation it can be seen that the net force on this body and therefore the acceleration points upwards. (c) Permanent magnet with drawn field lines. (d) Ray diagram for a convex mirror. (e) World lines for light travelling right and left and two other particles with smaller velocities. Reproduced from [4]*.(f) Faynman diagram representing interaction of electron and positron* [6].

2. The use of bar charts in problem solving in physics

Constancy versus conservation

It is very important to understand the difference between the constancy and conservation of a physical quantity, because these two terms are often confused. If you read carefully works of great physicists, like Enrico Fermi, they very clearly differentiate between these two terms in the following way.

Constancy means that the quantity does not change in a particular system, remains constant. In other words, that means that the system is isolated. If the system is not isolated and if the quantity is conserved, that means that the change of the quantity is always predictable and that we can always find another system for which this quantity is constant [5].

Energy is an example of a conserved quantity and volume is an example of quantity that is not conserved: if you mix 1litre of water and 1 litre of alcohol you get less than 2 litres of mixture.

2.1 Work-energy bar charts

It is of utmost importance that every time we are using the energy approach we always first choose (or define) a system and the initial and final state. Failure to do this systematically in every problem is one of the biggest deficiencies of our high-school and university physics and one of the main reasons why students have difficulties with this topic.

Then we analyse the processes using the following reasoning: total energy of the system is the sum of different forms of energy (some are only positive or zero (such as kinetic energy K) and some can be either positive or negative values (such as gravitational potential energy U_g)). If the two interacting objects are inside the system then we describe their interaction with an appropriate form of energy. As said before, a conserved quantity is constant in an isolated system. Therefore, when we are describing a work-energy process in an isolated system, the energy in the initial state must be of the same magnitude as the energy in the final state.

The important advantage of this approach is that it is a universal tool that can be applied to any situation without need for introducing exceptions (such as saying "…work done by all forces except the weight…"). This advantage is most evident in treating cases that involve gravitational potential energy as illustrated in the following example.

Consider an apple, which is initially at rest on the tree and then falls dawn on the floor. Final state is the moment right before it hits the floor.

First, we make a sketch with initial and final state of the problem, choose our system - the apple and the Earth and circle objects in the system as shown in Figure 3a.

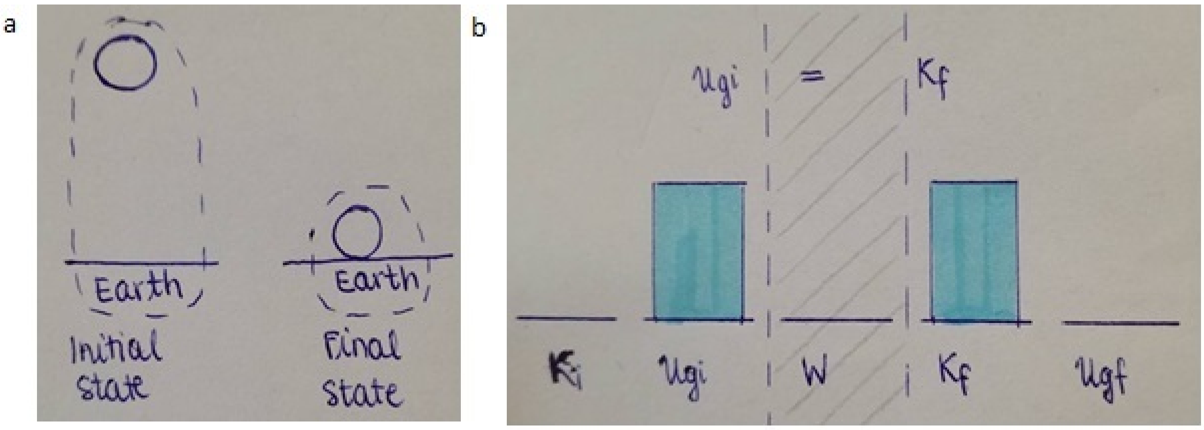

Figure 3: *(a) Sketch of a falling apple. (b) Bar chart and mathematical description for this problem.*

Then we construct a bar chart (Figure 3b). We draw a bar for every type of energy that is present in the system and one bar for representing work in case external objects do work on the system. The space where we draw the bar for work is shaded or separated with a (hatched) line to show that work is not a property of the system. In the initial state an object has gravitational potential energy (assuming we define zero of potential energy on the floor). At the end, the object only has kinetic energy. Since no external forces were exerted on the system, the work is zero (therefore the system is isolated). Bar charts help us to write mathematical representation of this process, as seen above the bar chart in Figure 3b.

Now we have been concerned with an isolated system, but if we do not have an isolated system and interacting objects are inside and outside the system, then we describe the interaction between these objects as work done on the system or heating.

When the system is not isolated, initial energy of the system plus the work done on the system must equal the final state energy. Therefore the change of the energy of the system can be predicted and that is the basis for constructing bar charts.

Now, we consider the identical problem as before, but this time we take the apple as a system (Figure 4a). In this case, we can not talk about gravitational potential energy, because Earth is not a part of the system. In this case Earth is an external object that is exerting force on the apple and therefore is doing work on the system. In the initial state the energy of the system (apple) is zero and in the final state the apple has kinetic energy as a result of work done by Earth on the system (recall that in this case the system is not isolated but energy is still conserved). Again we proceed by writing a mathematical representation as seen above the bar chart in Figure 4b. Therefore, a different choice of a system may result in a different bar chart.

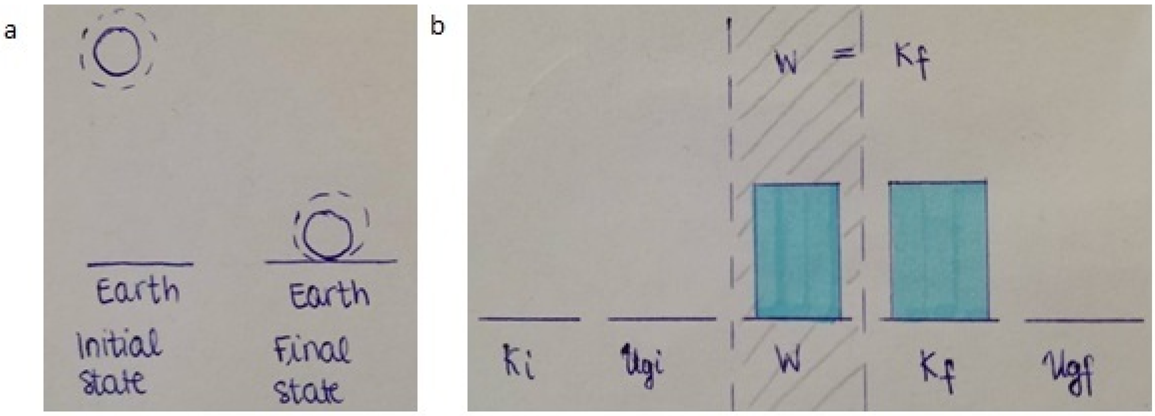

Figure 4: *(a) Sketch of a falling object. Here we should notice that the Earth is not included in our system as it was in Figure 3a. (b) Bar chart (notice that work done on the system is not zero) and mathematical description for this problem.*

Work-energy bar charts can be used in the mechanic courses as seen in the previous example and also in thermodynamics courses and modern physics.

Work-energy bar charts are helpful also when solving problems related to rotational motion. We know that kinetic energy of a rotating object can be written as $K_r = 1/2I\omega^2$. Imagine we have a ball at the top of the hill and it is initially at rest (Figure 5a). Then it starts rolling downhill and in the final state it has both translational and rotational kinetic energy. We take a ball and the Earth for our system and represent our situation with a bar chart (Figure 5b) to easier formulate mathematical description (see the equation above the diagram).

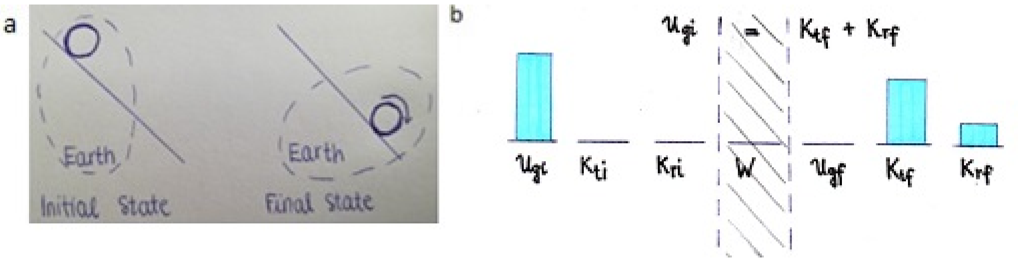

Figure 5: *(a) Sketch of a ball rolling downhill. (b) Bar chart for this problem and mathematical representation.*

When applying the first law of thermodynamics, we can also use work-energy bar charts as a visual representation. These bar charts are similar to the ones mentioned earlier, but we add another process of energy transfer – heating and another type of energy - internal energy. In introductory physics examples the kinetic and the gravitational potential energy of the system usually do not change and that is why we do not draw bar charts for these two quantities. Therefore the energy that can be different at the end of the process is internal energy. This means thermal energy or/and internal potential energy. In bar chart seen in Figure 6c, the space where work and heat transfer are represented is shaded, because none of them is a property of the system. On the left side of the chart we draw these two quantities and on the right side we draw the change of the internal energy (Figure 6c). Therefore the sum of quantities on the left must equal the change of the internal energy of the system [4].

Consider a simple isothermal expansion. First, we draw a sketch of the experiment and choose our system (Figure 6a), then we can represent the process with a graph (Figure 6b) and finally with bar chart (Figure 6c). If we have a negative quantity, we draw bar lower than the zero line. The bar chart representation allows us to write mathematical description of the problem (see the equation above the diagram).

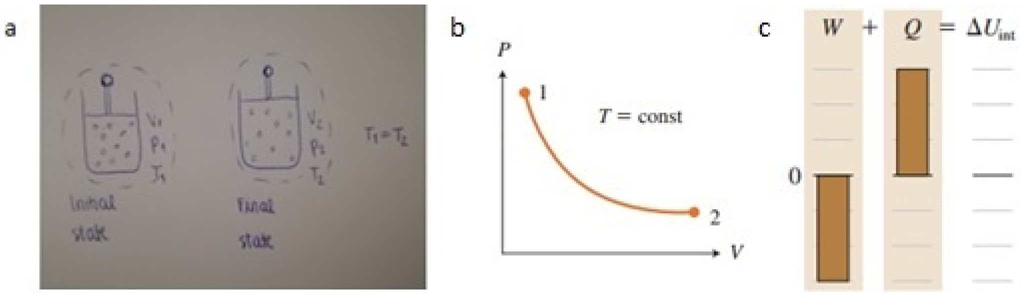

Figure 6: *(a) Sketch of an isothermal process. (b) Graph for an isothermal process. (c) Bar chart for such process and mathematical description* [4].

Like said before, bar charts are a useful representation also in modern physics. Photoelectric effect is one of phenomenon that can be presented and better understood with the use of bar charts. From now on, while we are discussing photoelectric effect, our system will be the nucleus and the electron as shown in Figure 7b. The electric potential energy of a bound electron (in a metal) is lower than zero. Such electron also has kinetic energy. As we know, the electron does not leave the metal if extra energy is not added. That is why the electron's magnitude of kinetic energy needs to be smaller than the magnitude of the previously mentioned electric potential energy. Therefore, the energy of the whole system is negative. The smallest energy that needs to be added in order to remove the electron from the metal is called the work function ϕ. For example, the energy of electrons can be increased by UV light (and sometimes visible light) absorption. If electron accepts more energy than ϕ, then it leaves the metal, for example a cathode, with a kinetic energy larger than zero. After we create a bar chart as seen in Figure 8c, we can write the mathematical description of the problem

$$K_{eli} + U_{qi} + E_{light} = K_{elf} + U_{qf}.$$

We can see a sketch of such experiment in Figure 7a. Because anode is the negatively charged electrode and cathode is the positively charged electrode, there is an electric field between them. When the electron leaves the cathode it has zero electric potential energy. When flying towards the anode, it is losing kinetic energy and gaining electric potential energy. If the kinetic energy is large enough, electrons reach the anode, despite the electric force pushing them away from it. That is how we obtain electric current, which can be measured.

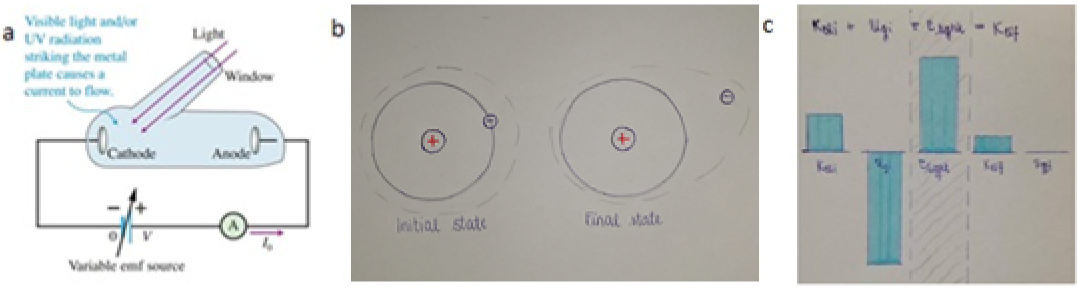

Figure 7: *(a) Sketch of a photoelectric effect* [4]. *(b) Sketch of an atom with a bound electron (initial state) and without the bound electron (after photoelectric effect – final state). (c) Bar chart representing photoelectric effect and mathematical description.*

We can use work-energy bar charts also to represent physical process in nuclear physics. Now we will take a look of what happens when we have nuclear particles (in our example two protons and two neutrons) - which we will take for our system - if they are far away at first and then they are brought together and nuclear fusion happens. In this example it is reasonable to observe what is happening with the electric potential energy and the nuclear potential energy of the system, assuming that kinetic energy is zero in the initial state. Electric potential energy (U_{qi}) is zero at first, since the nuclear particles are far away at the beginning. If we want to put protons closer, we need to add extra energy, for example, we can do work on the system. In order for fusion to happen, the work needs to be added, but only until the sum of positive electric potential energy and negative nuclear potential energy reaches its maximum. Then, as the protons get even closer, the magnitude of (negative) nuclear potential energy increases faster than the magnitude of electric potential energy. Therefore, the magnitude of the nuclear potential energy in the final state (U_{nf}) is bigger than the magnitude of the electric potential energy (U_{qf}). We can see that in the final state, in order for barchart to be balanced, we are missing some type of energy. That energy is kinetic energy and is usually released through heating. From the bar chart we can see, that we needed to add some energy for fusion to happen, but at the end we got a way bigger released energy. Fusion is an example of an exothermic reaction (Figure 8) [4].

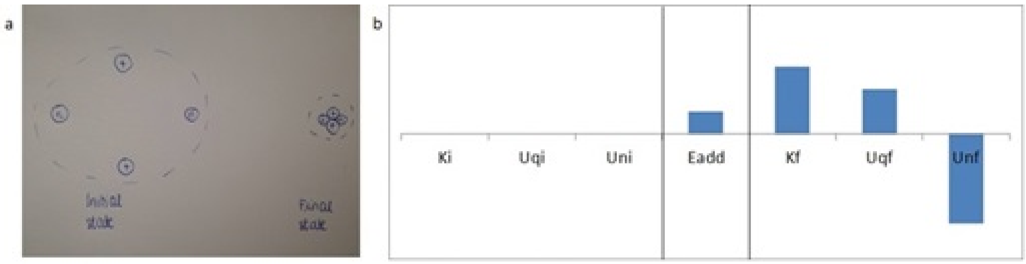

Figure 8: *(a) Sketch of two protons and two neutrons before and after nuclear reaction. (b) Bar chart for this nuclear reaction.*

2.1.1 More complex example of using energy bar charts

Let's see another example from everyday life, which is more complex and where you will see that we can use bar charts not only to represent what we already know, but also to solve problems or evaluate the solution.

Consider a person walking up the stairs. For the system we take a person, the Earth, the stairs and the surrounding air (Figure 9).

Figure 9: *a) Sketch of a person, walking up the stairs with labelled initial and final state and circled system. b) Bar chart representing this process.*

We will assume that the person is moving with a constant speed all the time and we will look at average values over several steps. At the beginning we have initial kinetic energy K_i, and zero gravitational potential energy U_{gi}, if assuming we define zero potential energy at the bottom of the stairs. Since no objects from the environment interact with the system, the work is zero.

In the final state the person is still moving with the same speed higher above Earth, so the kinetic energy is the same as before and potential energy is larger than before. Because the system is isolated we see from our bar chart in Figure 9b, that we are missing some type of energy to balance the left and right side. What remains is only internal energy. Because we are dealing with a live organism it is wise to split the change of the inertial energy into two components, the change of chemical energy ΔU_{ch} and the change of thermal energy ΔU_{th}. Chemical energy changes due to metabolism, where molecules with higher energy values decompose to molecules with lower energy values. Therefore in our case, the chemical energy decreases, so the change is negative. The change of thermal energy is related to the change in temperature of the system. Now we see: If we want the bar chart to be balanced, the change of the thermal energy must be positive which is consistent with the fact that we get warm while walking up the stairs (by-product of metabolism) and due to inelastic deformations of parts inside the system.

Let's see what happens, when we walk down stairs (we assume the person is moving at the same speed as before and descent for what was final state to initial state in previous example). Our system is the same as before (see Figure 10a).

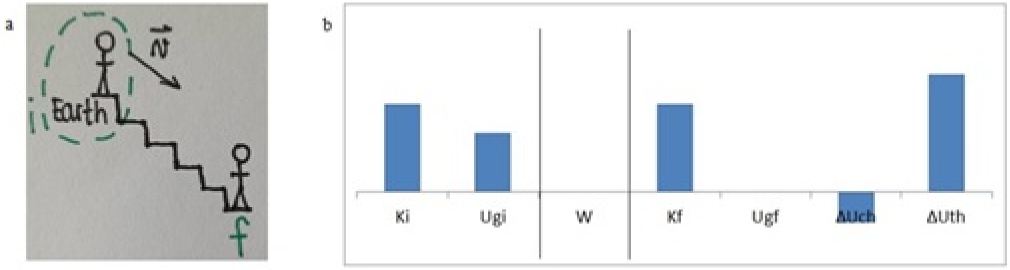

Figure 10: *a) Sketch of a person, walking down the stairs with labelled initial and final state and circled system. b) Bar chart representing this process.*

In the initial state, we have kinetic energy, since we are moving with a constant speed. Our gravitational potential energy is positive and no work is done on the system, because our system is isolated. In the final state the kinetic energy is of the same magnitude and gravitational potential energy is zero. Knowing that walking down is easier than up (with the same speed) we expect the change of chemical energy to be negative but in magnitude smaller than in the previous example. And here comes the surprise: in order to balance the bar chart seen in Figure 10b, we see that the change in thermal energy when walking down the stairs has to be larger than when walking up the stairs! When we walk down stairs we need to stop ourselves from falling down all the time: in every step we accelerate and then slow down by hitting the floor with our shoes. During this process, the floor, the shoes and even our joints undergo inelastic deformations and thus get warmer more than in the previous case.

2.2 Impulse-momentum and rotational momentum bar charts

Bar charts are a useful representation also when we discuss other quantities, for which conservation laws apply, for example momentum and rotational momentum. Impulse-momentum bar charts are similar to work-energy bar charts, but because momentum is a vector quantity, we represent laws of conservation separately for each component with a special bar chart. If the system is isolated, the sum of initial momenta of the objects is the same as the sum of the final momenta of the objects. If the system is not isolated, we need to consider external impulse, which changes the momentum of the system. The role of the work done on the system in work-energy bar charts is equivalent to the role of the impulse in impulse-momentum bar charts and therefore the impulse column is also shaded. Consider a simple example of collision of two carts that are the only objects in the system. In the initial state the carts are moving towards each other and the one on the left is moving faster. In the final state the carts are stuck together and are moving with the same speed. First we draw a sketch, then create a bar chart (in this case only for x component) and at the end write a mathematical description of the problem, similar as before in work-energy bar charts (see Figure 11) [4].

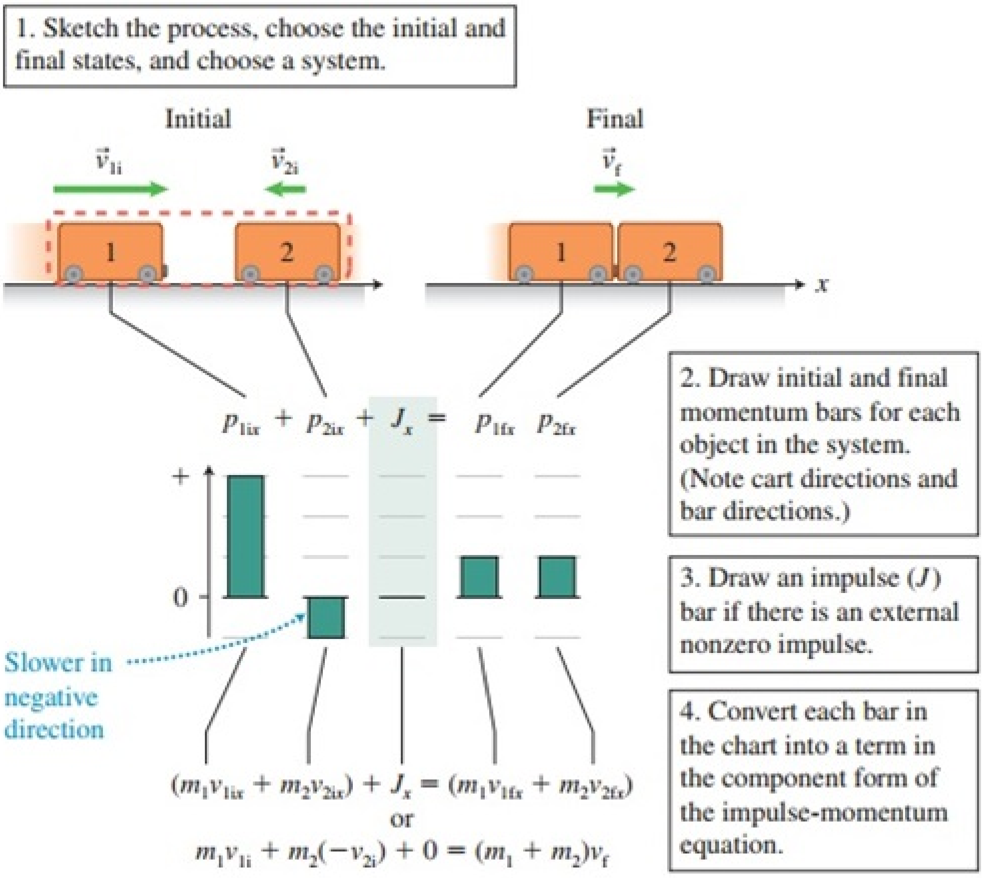

Figure 11: *Sketch, bar chart and mathematical description of a problem (two objects colliding)* [4].

Bar charts can also be used when working with rotational momentum. Such bar charts can be compared to impulse-momentum bar charts. Rotational momentum plays the role of momentum and rotational impulse plays the role of impulse. Bar chart of rotational impulse is drawn in the shaded area and is non-zero only if the system is influenced by external torques.

3. Conclusion

The article describes the benefits of using different representations in solving physics problems in various topics. The emphasis is on the use of bar chart representations. The introduction of the article shows that experts solve physical problems using different representations, while novices mostly only use sketches and mathematical descriptions, which may not allow them to develop deeper understanding of physical concepts. The introduction also shortly discusses some results of Physics Education Research and describes different representations: dot and motion diagrams, graphs, force and ray diagrams, field lines, spacetime diagrams and Faynman diagrams.

The second part of the article more thoroughly describes one more representation and that is a bar chart. It shows how they are constructed and in what physics fields they are most commonly used. The article also describes a more complex example of a problem where bar charts are used to solve the problem.

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