FAZNO KONJUGIRAJOČA OGLEDALA

ŽIGA PUŠAVEC

Fakulteta za matematiko in fiziko
Univerza v Ljubljani

Po odboju od fazno konjugirajočega ogledala žarek točno sledi poti, ki jo je opravil pred vpadom - ta isti nepričakovan učinek bi videli, če bi potovanje valovanja zavrteli nazaj v času. Pojav se imenuje fazna konjugacija in ga izkoriščamo za odpravo aberacije optičnih elementov na področjih visokoenergijskih laserjev, optičnih komunikacij in optičnih vezij. Je posledica tretjega reda nelinearnega optičnega odziva snovi. V članku sta predstavljena dva glavna mehanizma za dosego le tega - degenerirano mešanje štirih valovanj in povratno stimulirano Brillouinovo sipanje.

PHASE CONJUGATE MIRRORS

A Phase Conjugate Mirror reflects an incoming wave so that the reflected wave exactly follows the path it has previously taken - this can be interpreted as rewinding the time. This counterintuitive phenomenon has many interesting applications in correction of aberration that appear due to imperfect optical elements in fields of high energy lasers, optical communications and optical circuits. The origin of the phenomenon lies in the third order nonlinear optical response of matter. Two main mechanisms are discussed: degenerate four-wave mixing and stimulated Brillouin backscattering.

1. Introduction

The basic concept of light reflection from metallic and dielectric mirrors is well established. The angle between the reflected wave and the normal to the surface is the same as the angle between the incident wave and the normal. Light reflects back to its origin only for a wave that is perpendicular to the surface. This is described by The Snell's law and has been known for a long time [1].

In the late 1970s a new and very different kind of a mirror has been invented, based on the principles of third-order nonlinear optics. If one looked at himself in such mirror, one would see absolutely nothing[1]. This is a consequence of the fact that reflection from such a mirror has a different physical origin compared to a usual one. Due to nonlinear phenomena, the phase of reflected light is the exact conjugate of the incident one. Such mirrors are called *phase conjugate mirrors* (PCM). The main consequence of phase conjugation is that light, reflected from such mirrors, exactly follows the path it had made before hitting the mirror, matching in phase at all points along the way [2]. This basic concept has many useful applications in fields of high energy lasers, communication signals and adaptive optics [3].

2. Basic Concept of Phase Conjugation

To discuss various applications of PCM, the exact knowledge of underlying non-linear mechanism(s) is not required. So we will discuss it later in *Section 4*, when the context will already be well established.

The general concept of phase conjugation is very simple – the phase of the incident light gets conjugated on reflection. To describe this effect let us define[2] the incident wave travelling in the direction $\hat{e}_z$ with

$$E_3(x,y,z;t) = E_0(x,y,z)\ e^{i(\Phi(x,y,z)-\omega t)} + c.c.\ , \tag{1}$$

---

[1] In fact, one does see a little bit of light, but only the few rays that reflect from his eyes directly onto the mirror. The intensity of such light, however, is very low and difficult to detect.

[2] The choice of indices *3* and *4* will become clear later in *Section 4.3.*

where $\Phi(x,y,z) = kz + \varphi(x,y,z)$ is the dependence of the space component of the wave's phase. The $\varphi(x,y,z)$ describes the distortion of the wave front (its phase) by the medium while passing through it. We can now rewrite this as

$$\begin{aligned} E_3(x,y,z;t) &= E_0(x,y,z)\ e^{i(kz+\varphi(x,y,z))}\ e^{-i\omega t} + c.c. \\ &= A_3(x,y,z)\ e^{-i\omega t} + c.c.\ , \end{aligned} \tag{2}$$

where we have defined A_3 as the complex amplitude of the incident wave. Now we can write the reflected wave in the same way, while taking into account also that its phase gets conjugated. This means that the propagation vector changes its sign as $\mathbf{k} \to -\mathbf{k}$ and the same happens for its phase distortion: $\varphi \to -\varphi$. The reflected wave is then

$$\begin{aligned} E_4(x,y,z;t) &= E_0(x,y,z)\ e^{i(-\Phi(x,y,z)-\omega t)} + c.c. \\ &= E_0(x,y,z)\ e^{i(-kz-\varphi(x,y,z))}\ e^{-i\omega t} + c.c. \\ &= A_4(x,y,z)\ e^{-i\omega t} + c.c.\ , \end{aligned} \tag{3}$$

where we have again defined A_4 as the complex amplitude of the wave. Comparing the two expressions we see that $A_4 = A_3^*$. To better understand this, it is worth writing down the real parts of the waves

$$\begin{aligned} Re[E_4(x,y,z;t)] &= 2\ E_0 \cos(kz + \varphi(x,y,z) + \omega t) \\ &= 2\ E_0 \cos(-(-kz - \varphi(x,y,z) - \omega t)) \\ &= 2\ E_0 \cos(kz + \varphi(x,y,z) - \omega(-t)))\ . \end{aligned} \tag{4}$$

Thus we see

$$Re[E_4(x,y,z;t)] = Re[E_3(x,y,z;-t)]\ . \tag{5}$$

From this, we can interpret the reflected wave as the incident wave, reversed as a wave going back in time, exactly following the path it had previously took. If we illuminate such mirror with light from a point source, the reflected light will not expand after reflection, but rather focus back to its source [4, 5].

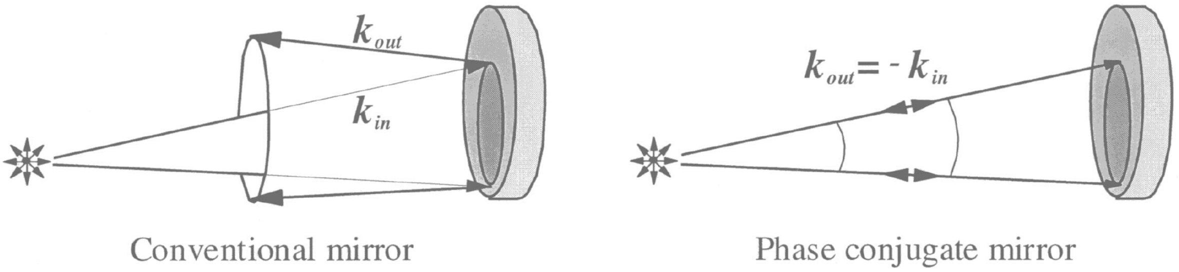

Figure 1. The wave reflects differently from PCM than from an ordinary mirror. After reflection from a PCM a previously diverging beam converges back to its origin. *Reproduced from* [5].

3. Applications of Phase Conjugate Mirrors

Phase conjugation gives us a very different and counterintuitive experience with reflection and as such offers various new and interesting applications, which will now be presented.

3.1 Compensation of Aberrations

One of the main applications of PCMs is in compensation for imperfect optical elements. While travelling through an non-ideal medium, different rays of light take different optical paths. This causes a distortion of their plane wavefront. The phase distortion caused by the medium is described with $\varphi(x, y, z; t)$. Because of the inversion of the wavefront at reflection from PCM, we can view the conjugated wave E_4 as going back in time. This means that during the second pass through the distorting medium, the phase distortion gets repaired, and when it re-emerges from the medium, its wavefront is again plane [6]. This concept is the basis of the applications that are going to be discussed here. It is worth emphasising that there exist two main limitations to this process: first, in the time it takes the light wave to travel there and back through the distorting medium, $\varphi(x, y, z; t)$ must not change significantly. Secondly, light must not alter the physical properties of the medium while passing through it [6].

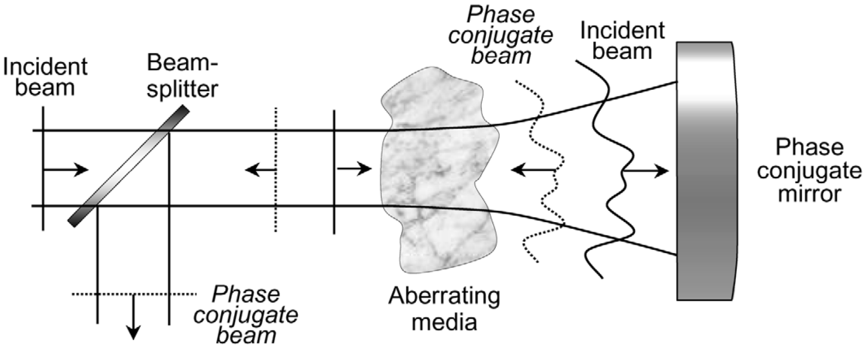

Figure 2. The originally plane wave, that is reflected from PCM, reemerges as an undistorted plane wave when exiting the distorting medium. *Reproduced from* [7].

The fundamental example of application is in long range communications. If we send a light signal to a detector relatively far away, its wavefront will get distorted due to the density variations in the atmosphere. If we send this already distorted signal back, it will get additionally distorted. But if a PCM is used for reflection, it remains almost undistorted. The original receiver is also automatically targeted, as discussed in *Subsection 3.3.* However, information in such system should be stored in the signal's intensity[3], not in its phase [4].

3.2 Optical Fibres

The common way of information transfer nowadays is via optical fibres. The main limitation of optical fibres are different types of dispersion. While some of them can be reduced by fibre design[4], there are still some which cannot be avoided, like dispersion due to multiple possible wave modes present in fibres. With a PCM we can reduce some of them.

In multimode optical fibre, dispersion is a consequence of the fact that different allowed wave modes

---

[3] Amplitude or the position of the pulse [3].

[4] For example, we can compensate the effect of the chromatic dispersion with the waveguide dispersion [14].

travel with different group velocities. The pulse length thus expands. One way to reduce such effect is by using single mode optical fibres. Another way is by using PCMs. If we put a PCM on one end of the optical fibre, the wave's phase will get reversed on reflection. When such reflected wave is sent back trough an identical optical fibre, its dispersion will get reduced while travelling back. The previously faster mode is now slowed down [3]. This enables communications with a distant source while in return receiving an undistorted and low-noise signal. Information is again stored in the intensity of the signal (amplitude, length of the pulse). On the receiving end the information can be manipulated using electrooptic effect. According to Pepper et al., such manipulation causes negligible loss of intensity [3]. Thus, two way communication with very low level of noise can be achieved.

It is worth emphasising that due to PCM, not only single mode, but also multimode optical fibres can be used for communication, which can significantly increase the amount of transferred data. Again, Pepper et al. conclude that the upper data bandwidth limit in such optical fibres will be determined by the quality of electronics, not by the quality of PCM [3].

3.3 High-Intensity Lasers

The aberration compensation can be very effectively used in high-intensity lasers. Constructing a good laser is an art by itself, but the main components are fairly simple: the amplifying medium, through which the energy is pumped into the system (by the principle of *inverted population*) and a resonator consisting of two mirrors, which allow light beams to pass through the amplifier multiple times [1]. One of the engineering limitations is that while passing through the amplification medium, the beam gets diffracted/distorted because of refractive index changes with space due to inhomogeneous material and thermal fluctuations. This can be solved by replacing one of the resonator mirrors with a PCM. This results in a beam, that has been amplified twice, but has not diverged during the amplification [10]. The first experimental realizations of this concept in 1970s have proven usefulness of this concept when a decrease of beam divergence from 2,5 to 0,15 mrad was achieved by Pepper et al. [3].

This kind of application also solves another problem. To achieve stable operation of a standard laser, only certain curvatures of mirrors are allowed [14]. With a PCM, this is no longer needed, since light beam does not diverge when double passing through the amplification medium [2]. As such, PCMs can be used to create new types of high energy lasers that can achieve better beam quality and stability than ordinary ones [6].

In practice, PCMs are most frequently found in *Master Oscillator Power Amplifier* (MOPA) laser system [5], used to achieve high intensities of lasers in several steps. In ordinary lasers, output light intensity is limited by the fact that the effective power per volume of the amplification medium is upward-limited. It is then more effective to have several smaller amplifiers/resonators coupled together. The PCM is here used to coherently combine a set of parallel amplifiers, while compensating for individual phase shifts between individual beams [3]. Experimentally, H. J. Eichler and O, Mehl have shown that the power of a base oscillator has been increased from 1 W to 130 W by using this technique [5].

3.4 Laser Tracking/Automatic Alignment

PCMs can also be used for alignment of laser beams. It has been suggested that it could be useful for targeting of fusion pallets in fusion reactors, where there exists a problem of focusing light in to the reactor core, as it otherwise gets too distorted before even reaching the target [10].

The general concept is again fairly simple. If a (slowly moving) target is illuminated with a low energy laser, light will reflect from it in all directions. A small part of intensity of light will reflect in the direction of the laser tracking system. This light will first pass through a strong amplifying medium and then reflect from a PCM. Because of the basic properties of PCMs, the reflected light will directly return to the target without any divergence, the only difference being that it will twice pass the (strong!) amplification medium. Thus, a very strong laser beam can be automatically sent on a target, which would otherwise be hard to hit [10].

3.5 Other Applications

Only some most basic applications have been discussed in order to illustrate the concept of phase conjugation in optical systems. The use of PCMs is however not limited to these examples - many additional applications can be found in fields such as image projection [2], spectroscopy [2], photolithography in production of silicon chips [11], pattern recognition [7], parallel processing in optical computing [3] and other similar.

4. Theoretical Description of Phase Conjugation

The physical process of phase conjugation results from different physical phenomena. The two most important ones, degenerate four-wave mixing and stimulated Brillouin backscattering will now be presented, but first some general context has to be established about nonlinear optics, as well as the concept of holography.

4.1 Nonlinear Optics

The discovery of the nonlinear optical response is a fairly recent discovery [14], as high light intensities are required. This can only be achieved with lasers, which have only been available for the last 50 years or so. The nonlinear response is similar to the one in magnetic systems. For low intensities of external excitations we can approximate the response of the medium (magnetization) $\mathbf{M}$ to be linear with the excitation $\mathbf{H}$: $\mathbf{M}(\mathbf{H}) \approx \chi_m \mathbf{H}$. In optics, we can do the same for dielectric polarization $\mathbf{P}(\mathbf{E}) \approx \chi_e \ \mathbf{E}$. In the linear approximation the *susceptibility* χ_e is

$$\chi_e = \epsilon - 1 \ . \tag{6}$$

This is of course only true for low intensities of incident light. To describe the nonlinear response at higher intesities we must define several susceptibilities of different ranks

$$\begin{aligned} \mathbf{P} &= \mathbf{P}_{linear} + \mathbf{P}_{nonlinear} \\ &= \epsilon_0 \ \chi^{(1)} \ \mathbf{E} + \epsilon_0 \ \chi^{(2)} \colon \mathbf{EE} + \epsilon_0 \ \chi^{(3)} \ \vdots \ \mathbf{EEE} \ + ... \end{aligned} \tag{7}$$

A selected component of dielectric polarization $\mathbf{P}$ is written as

$$(P_{nonlinear})_i = \epsilon_0 \ \chi^{(2)}_{ijk} \ E_j E_k + \epsilon_0 \ \chi^{(3)}_{ijkl} \ E_j E_k E_l + ... \ . \tag{8}$$

It is worth mentioning that $\chi^{(2)}$ is non-zero only for materials without a centre of inversion, while $\chi^{(3)}$ is always present. Because of this, third order nonlinear phenomena are observable in all states of matter – hence PCMs can be constructed using any of them [3].

4.2 Holography

One of the limitations of classical light detection is that only information about the amplitude of the wave is detected and information about phase is lost. An example of this is classical X-ray crystallography. This has inspired the invention of holography. The hologram is created in two separate steps. First, the initial beam is split into two parts. One part, so called *reference wave* illuminates the photoactive film directly. Second, *signal wave*, is shone onto the selected object, from which it reflects. The reflected wave has a modified wavefront, which can be described by a varying phase of the wave. When the two waves meet at the point of the photoactive film, they interfere. Due to their different phases, a diffractive grating appears in the film emulsion, which stores the information about the object. The second step consists of reading this information by using a third *reconstructive wave*, which diffracts from the grating. By observing this diffracted wave the 3D image of the original object becomes visible [8].

This is the origin of PCM. It comes from the discovery that if one illuminates the film grating with a beam that has the same characteristics as the reference beam, but travels in the opposite direction, a real wave is produced, that is the conjugate of the original one [2].

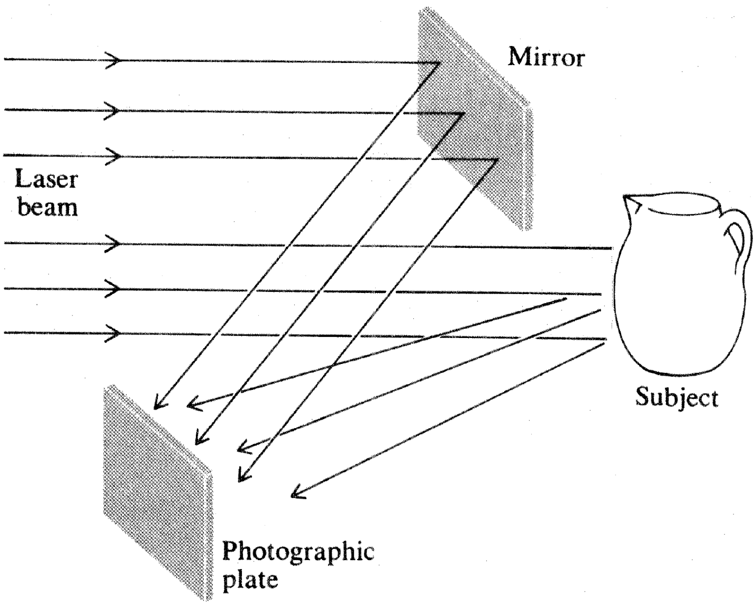

Figure 3. A typical setup for recording of a hologram. The reference wave illuminates the photographic plate directly and the signal wave is reflected onto it from the observed subject. Due to different phases the two interfering waves form a diffractive grating on the photographic plate - a hologram. *Reproduced from* [15].

4.3 Degenerate Four-Wave Mixing

The practical use of the above mentioned method is limited due to its nature as a two-step procedure. The next step, which made the phase conjugation useful in practice, was that the refractive grating was successfully made to change in real-time. This was achieved with use of third order nonlinear effects. Information from incoming signal wave could thus be stored in the grating by the wave coupling of a signal wave with one of the pump waves. At the same time, this information could be read from the grating by the other pump wave, whose scattered light exited the medium as a wave, which was the exact phase conjugate of the original[5]. This is the main concept behind the *real-time holography*, better known as *degenerate four-wave mixing* (DFWM) [6].

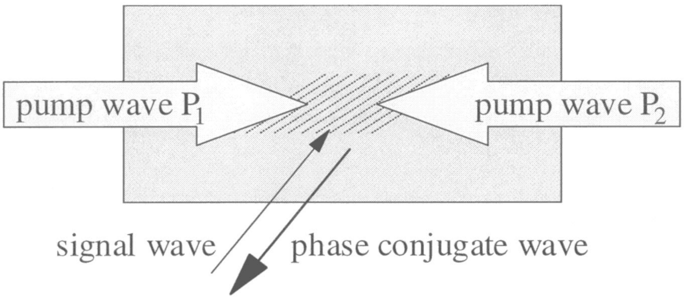

Figure 4. The most common scheme for DFWM. We will describe pump waves with E_1 and E_2, the signal wave with E_3 and the phase conjugated wave with E_4. The dashed lines represent the diffraction grating that is formed due to nonlinear response of matter. *Reproduced from* [5].

The most common setup for DFWM is shown in the *Figure 3*, with three incoming waves E_1, E_2 and E_3 and one outgoing wave E_4. It is important that the *pump waves* E_1 and E_2 are propagating in opposite directions ($\mathbf{k}_1 = -\mathbf{k}_2$), the amplitudes of the pump waves are much larger than the one

---

[5] We now see that PCM is not an actual mirror, since the light does not really reflect from it. The light that we perceive as reflected actually originates from the pump waves, which scatters on the diffractive grating.

of the *signal wave* E_3 and that all four waves have the same frequency ω. We will see that from this conditions one can get the outgoing *phase conjugated wave* E_4 [4].

To obtain the general solution for four-wave mixing we have to solve the nonlinear wave equation

$$\nabla^2\mathbf{E} + \epsilon\frac{\omega^2}{c_0^2}\mathbf{E} = \mu_0\frac{\partial^2\mathbf{P}_{nonlinear}}{\partial t^2}\ . \tag{9}$$

In general, the outgoing wave is given by [8]

$$(P^{(3)}_{nonlinear})_i = \epsilon_0\ \chi^{(3)}_{ijkl}(\omega_1,\omega_2,\omega_3,\omega_4)\ E_j(\omega_1)E_k(\omega_2)E_l(\omega_3) + c.c.\ . \tag{10}$$

In the most general case, 384 terms appear in this expansion [2]. This number can be vastly reduced, if we take into account the specifics of DFWM. The orientation of pump waves ($\mathbf{k}_1 = -\mathbf{k}_2$) reduces the problem to one dimension and ensures phase matching at all times [8]. The outgoing frequency is then $\omega_4 = \omega_1 + \omega_2 - \omega_3$. If we now consider the degenerate case where $\omega_1 = \omega_2 = \omega_3 = \omega$, we see that also $\omega_4 = \omega$. By choosing the same polarization for all four waves we can reduce the tensor $\chi^{(3)}_{nonlinear}$ to only one element $\chi^{(3)}$ [8]. By assuming that the amplitudes of the pump waves are much greater than of the amplitude of the signal wave and by inserting the ansatz for plane waves into *Eq.(9)*, we obtain [7]

$$\frac{\mathrm{d}E_4}{\mathrm{d}z} = \ i\ \frac{\omega_0}{2}\ \sqrt{\frac{\mu}{\epsilon}}\ \chi^{(3)}E_1E_2E_3^* \tag{11}$$

$$\frac{\mathrm{d}E_3}{\mathrm{d}z} = -\ i\ \frac{\omega_0}{2}\ \sqrt{\frac{\mu}{\epsilon}}\ \chi^{(3)}E_1E_2E_4^*\ . \tag{12}$$

These two equations describe the process of phase conjugation by DFWM in PCMs. We describe the effect of the medium as well as of the two pump waves with introducing a new constant κ, defined as

$$\kappa = \frac{\omega_0}{2}\ \sqrt{\frac{\mu}{\epsilon}}\ \chi^{(3)}E_1E_2\ . \tag{13}$$

This results in the spatial dependence of the refractive index, which generates the formation of the diffraction grating [8]. After some calculation we get the solutions to these equations [2, 14]

$$E_4(z=0) = E_3^*(z=0)\frac{\kappa}{|\kappa|}\tan(|\kappa|L) \tag{14}$$

$$E_3(z=L) = E_4^*(z=L)\frac{1}{\cos(|\kappa|L)} \tag{15}$$

where $\hat{e}_z$ is the direction of incident wave and L is the length of the nonlinear system. The $z=0$ indicates the beginning of the system and $z=L$ its end. This is how we phase conjugate an incoming wave using DFWM.

4.3.1 Reflective Coefficient

One of the interesting consequences of DFWM is that the reflective coefficient $R = \tan^2(|\kappa|L)$ can be larger than 100 %. This is because energy can be transferred from the strong pump waves to the signal wave. In experiments, the reflectivity up to 10.000 % was achieved [11]. This property can be used in designing lasers with a PCM [3].

4.4 Stimulated Brillouin Backscattering

One of the main disadvantages of DFWM is that we need two strong matching pump lasers, which can be hard to achieve in practice on a small scale. A different kind of process that can be used to solve this is *stimulated Brillouin backscattering.* In fact, it was through this process that the first real PCM was constructed in 1972 [3].

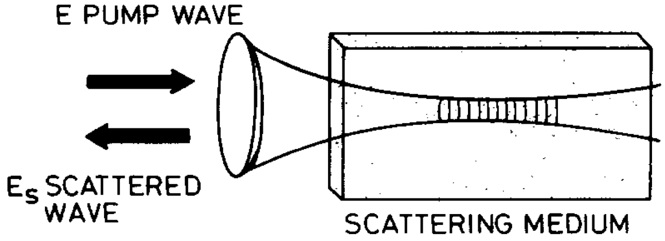

Figure 5. The general scheme of SBS. The incoming pump wave is scattered in the medium in all directions. The part of it that is reflected back interferes with the incoming wave and thus creates a positive feedback loop. Thus only the exactly backscattered wave is sufficiently amplified. *Reproduced from* [2].

Due to thermal fluctuations, acoustic waves (phonons) can be formed in matter [2]. This creates a moving diffractive grating, from which the incoming wave E_3 is Brillouin-scattered [8]. If it is of sufficient intensity, the backwards going beam E_4 then interferes with E_3 and creates even more sound waves due to the electrostrictive force[6] [6]. The trick for construction of PCMs is in successfully creating a positive feedback loop between these two processes [6]. This can be achieved at certain frequencies and leads to an exponential growth of E_4. Even though the initial wave scatters in all directions, it can be shown that the exactly phase conjugated wave gets amplified most [6].

This leads to a more practical approach for creating a PCM. The main disadvantage of this process is that the created wave is not ideal. Due to the reflection from a moving sound wave, some energy is lost to the phonons and the frequency of E_4 gets Doppler shifted [2]. Nevertheless, the reflectivity of such mirrors can reach 100 % [11].

5. Conclusion

Phase Conjugate Mirrors demonstrate an interesting example of diverse phenomena that can be achieved in the very rich field of nonlinear optics. It also serves as an example of how complex industry designs can originate from a single very simple concept. Inspired by this, one can imagine that several new inventions will be made in the coming years, which will alter our conceptions of how the modern technology will develop even further.

---

[6] This is a force that points in the direction of the increasing electric field in a dielectric material [6].

6. Acknowledgement

I would like to thank dr. Mojca Vifan for her help and support in the writing of this article.

REFERENCES

[1] B. E. A. Saleh & M. C. Teich. *Fundamentals of Photonics,* 2^{nd} *Edition.* John Wiley & Sons, Inc., USA, 2007.

[2] M.C. Gower. *The Physics of Phase Conjugate Mirrors.* Progress in Quantum Electronics, Vol. 9, p. 101-147, 1984.

[3] D. M. Pepper, David A. Rockwell & Gilmore J. Dunning. *Nonlinear Optical Phase Conjugation.* IEEE Circuits and Devices Magazine, Vol. 7, Issue: 5, p. 21-34, 1991.

[4] G. Grynberg, A. Aspect & C. Fabre. *Introduction to Quantum Optics.* Cambridge University Press, New York, 2010.

[5] H. J. Eichler & O. Mehl. *Phase Conjugate Mirrors.* Journal of Nonlinear Optical Physics & Materials, Vol. 10, No. 1, p. 43-52, 2001.

[6] B. J. Feldman, et al. *Through the Looking Glass with phase conjugation.* Los Alamos Science, Fall 1982.

[7] J. P. Huignard & A. Brignon. *Overview of Phase Conjguation; Phase Conjugate Laser Optics.* John Wiley & Sons, Inc, 2004.

[8] R. Guenther. *Modern Optics.* John Wiley & Sons, Inc., USA, 1990.

[9] M. Čopič, A. Petelin & M. Vilfan. *Fotonika.* FMF, 2018.

[10] Y. R. Shen. *The Principles of Nonlinear Optics.* John Wiley & Sons, Inc., USA, 2003.

[11] M. Gower. *Phase Conjugate Mirrors – Mirrors that reflect time.* Nature, Vol. 308, 8, 1984.

[12] P. Meystre & M. Sargent III. *Elements of Quantum Optics.* Springer-Verlag, Berlin, 1990.

[13] R. Kowarschik, et al. *Optical Measurements with phase-conjugate mirrors; Applied Physics B, Lasers & Optics.* Scpringer-Verlag, 1999.

[14] P.A. Franken, A.E. Hill, C.W. Peters & G. Weinreich. *Generation of Optical Harmonics.* Physical Review Letters, vol. 7, Issue 4, pp. 118-119, 1961

[15] G.R. Fowles. *Introduction to Modern Optics.* Dover Publication, Inc., New York, 1975