THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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In this article theoretical calculations and measurements of the anomalous magnetic moment of the muon (a_μ) are presented. First muon anomalous magnetic moment is defined. Then the Standard model contributions to a_μ are described, followed by the presentation of experiments that measure a_μ. Next, the discrepancy between theory and experiment is discussed. In conclusion, some of the new physics proposals that could solve mentioned discrepancy are considered.

ANOMALNI MAGNETNI MOMENT MIONA

Članek predstavlja teoretične izračune in meritve anomalnega magnetnega momenta miona (a_μ). Sprva je anomalni magnetni moment miona definiran. Nato so opisani prispevki standardnega modela k a_μ. Sledi predstavitev eksperimentov, ki so in bodo izmerili a_μ, ter razprava o vzrokih odstopanja med teorijo in eksperimentom. Na koncu so omenjeni prispevki nove fizike kot rešitev omenjenega odstopanja.

1. Introduction

The anomalous magnetic moment of a lepton a_ℓ is a dimensionless quantity. It can be computed as a perturbative expansion in the fine structure constant α in quantum electrodynamics (QED) and beyond. Besides a_e (the anomalous magnetic moment of the electron), a_μ is one of the most precisely measured quantities in particle physics [1]. Both measurements are high-precision tests of the Standard Model (SM) of particle physics [2].

If we use the most precise determination of the fine structure constant α, which does not depend on a_e, to calculate the theoretical prediction of the electron anomalous magnetic moment (a_e^{SM}), we see that latter is in good agreement with the recent extraordinary precise measurement a_e^{exp} [1]. Unlike a_e, a_μ^{exp} measured by Brookhaven National Laboratory (BNL) E821 experiment represents an interesting but not yet conclusive 3.6 sigma discrepancy from the Standard Model prediction (a_μ^{SM}). The latest E821 measurement was performed in 2001, but the upcoming Fermilab E989 experiment offers some improvements. The Muon g-2 collaboration at Fermilab aims to reduce the total uncertainty by a factor of four and expects to collect 21 times the E821 statistics. The first and the last results are expected to release in 2018 and 2020 respectively [2]. If the precision is improved and the central value stays unchanged, the difference between theory and experiment will rise up to 7.5 standard deviations [3]. This could be an even clearer sign of new physics beyond the Standard Model (BSM).

The higher the mass of the lepton, the more sensitive to BSM its anomalous magnetic moment is. This sensitivity to new physics scales with $(m_\ell/\Lambda)^2$ where Λ is the scale of new physics. While a_e is not so interesting, even though it requires to push QED calculations to higher orders, a_μ is much more attractive due to BSM sensitivity ratio $(m_\mu/m_e)^2 \approx 43000$. It is sensitive to all kinds of effects and thus forces theorists to predict new theories of physics beyond the Standard Model [1,2].

2. Magnetic moments

Consider the Pauli equation. This is the Schrödinger equation, which takes into account the interaction of the spin-1/2 particles with an external electromagnetic field

$$i\hbar\frac{\partial\hat{\varphi}}{\partial t} = \hat{\mathbf{H}}\hat{\varphi} = \left[\frac{1}{2m}(\mathbf{p}-e\mathbf{A})^2 + e\Phi - \frac{e\hbar}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}\right]\hat{\varphi}, \tag{1}$$

where

$$\psi = \hat{\psi} e^{-i\frac{mc^2}{\hbar}t} \quad \text{and} \quad \hat{\psi} = \begin{bmatrix} \hat{\varphi} \\ \hat{\chi} \end{bmatrix} \tag{2}$$

are Dirac four-spinors, $\hat{\varphi}$ is Pauli two-spinor, m is the mass of a particle, e is its electric charge, $\boldsymbol{\sigma}$ are Pauli matrices collected into a vector, $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$ is an external magnetic field and Φ and $\mathbf{A}$ are scalar electric and vector magnetic potential respectively. We are dealing with an equation that is the non-relativistic limit of the Dirac equation. The last term, which represents potential energy of a magnetic dipole in an external field, is the one we are interested in. It has the form of a magnetic interaction Hamiltonian $-\boldsymbol{\mu} \cdot \mathbf{B}$, so we can define the particle's intrinsic magnetic dipole moment

$$\boldsymbol{\mu} = \frac{e\hbar}{2m}\boldsymbol{\sigma} = \frac{e}{m}\mathbf{S}; \quad \mathbf{S} = \hbar\mathbf{s} = \hbar\frac{\boldsymbol{\sigma}}{2}, \tag{3}$$

where $\mathbf{S}$ is spin angular momentum and $\mathbf{s}$ is some kind of spin quantum number. For comparison: the orbital magnetic moment is

$$\boldsymbol{\mu}_{\text{orbital}} = \frac{e}{2m}\mathbf{L} = g_l\frac{e}{2m}\mathbf{L}; \quad \mathbf{L} = \mathbf{r} \times \mathbf{p} = \hbar\mathbf{l}, \tag{4}$$

where $\mathbf{L}$ is orbital angular momentum. Now we can construct the total magnetic moment

$$\boldsymbol{\mu}_{\text{total}} = \frac{e}{2m}(g_{\text{l}}\mathbf{L} + g_{\text{s}}\mathbf{S}) = \frac{m_e}{m}\mu_{\text{B}}(g_{\text{l}}\mathbf{l} + g_{\text{s}}\mathbf{s}), \tag{5}$$

where g_{l} and g_{s} are orbital and spin g-factors (gyromagnetic ratios) respectively and

$$\mu_{\text{B}} = \frac{e\hbar}{2m_e} \tag{6}$$

is Bohr's magneton. If we consider a lepton and thus use negative electric charge, then g_{l} and g_{s} are actually negative, $g_{\text{l}} = -1$ and $g_{\text{s}} = -2$. The latter is the famous result, which we get from Dirac or Pauli equation $g_\ell^{(0)} = 2$[a][b], but in the framework of quantum field theory (QFT), the value is slightly exceeding 2[c] [1].

If we neglect electrical fields, the quantum correction becomes a single number. This is the anomalous magnetic moment, defined as

$$a_\ell \equiv \frac{g_\ell - 2}{2}, \quad (\ell = e, \mu, \tau), \tag{7}$$

which we can get as a result of radiative corrections (RC) or sometimes called relativistic quantum fluctuations. We could say that the gyromagnetic ratio of a lepton is defined as the ratio of the magnetic moment and the spin operator in units of $\mu_0 = e\hbar/2m_\ell$

$$\boldsymbol{\mu} = g_\ell\frac{e\hbar}{2m_\ell}\mathbf{s}; \quad g_\ell = 2(1 + a_\ell). \tag{8}$$

From the last equation, we can easily distinguish between the tree level part and the *anomalous* part. One can also define anomalous magnetic moment as

$$a_\ell = \mu_\ell/\mu_0 - 1 = \frac{1}{2}(g_\ell - 2)\ [1]. \tag{9}$$

[a] It is a tree level contribution.
[b] From now on we will not deal with the sign of gyromagnetic ratio.
[c] a_ℓ is different for each lepton.

3. Derivation of radiative corrections

Remember, we are considering the interaction of a particle in an external magnetic field (1). If we now switch to quantum field theory, our Lagrangian should include the full QED interaction term

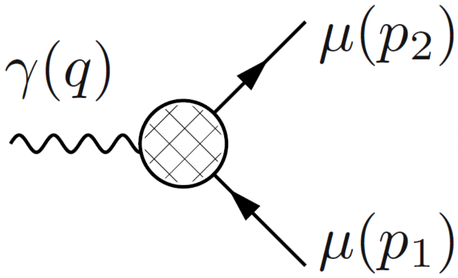
$$\mathcal{L}_{\text{int}}^{\text{QED}} = -e\bar{\psi}\gamma^{\mu}\psi A_{\mu}, \tag{10}$$

where the photon field is a part of the dynamics but has an external classical component A_{μ}^{ext}

$$A_{\mu} \to A_{\mu} + A_{\mu}^{\text{ext}}. \tag{11}$$

Therefore we have to add to our set of contributions to the matrix element for the scattering amplitude an additional external field vertex $-ie\gamma^{\mu}A_{\mu}^{\text{ext}}$ [1].

Because the magnetic moment is related to the interaction of spin particles with an external electromagnetic field, we simply have to take into account Feynman diagrams where a photon is interacting with fermion (in our case muon) (see figure 1). We can identify this diagram with the following expression



Slika 1. General Feynman diagram used to derive contributions to the muon anomalous magnetic moment. q is photon's, while p_1 and p_2 are muon's four-momenta. The shaded circle indicates the sum of the lowest-order muon-photon vertex and all amputated loop corrections. Reproduced with permission of F. Jegerlehner (2007) [1].

$$(-ie)\bar{u}(p_2)\left[\gamma^{\mu}F_{\text{E}}(q^2) + i\frac{\sigma^{\mu\nu}q_{\nu}}{2m_{\mu}}F_{\text{M}}(q^2)\right]u(p_1), \tag{12}$$

where $q = p_2 - p_1$, $u(p)$ is the Dirac spinor, $F_{\text{E}}(q^2)$ is the electric charge or Dirac form factor and $F_{\text{M}}(q^2)$ is the magnetic or Pauli form factor. We should also note that the matrix $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$ represents the spin 1/2 angular momentum tensor. Taking into account the static limit, where there is no momentum transfer $q^2 = (p_2 - p_1)^2 \to 0$, we have

$$F_{\text{E}}(0) = 1, \quad F_{\text{M}}(0) = a_{\mu}, \tag{13}$$

where the first equation is charge renormalization condition, and the second one is the prediction for a_{μ}, in terms of the form factor F_{M} [1].

The muon or any other charged lepton interacts electromagnetically via the photon and weakly with the heavy gauge bosons $W^{\pm}$, Z^0 [1]. It is surrounded by a thin soup of particles. If these virtual particles are lepton or quark particle-antiparticle pairs, then we are talking about vacuum polarisation. This pairs are effectively dipoles and make the vacuum a dielectric medium. Particles are also somewhat magnetic so they increase the gyromagnetic ratio to $g_{\mu} = 2(1 + a_{\mu})$ [4].

The small correction a_{μ} is a consequence of many contributions[d] which will be discussed in the following:

3.1 QED

The only relevant diagrams in quantum electrodynamics are the ones that contain photons and leptons. Contributions are calculated perturbatively as the expansion in powers of α/π.

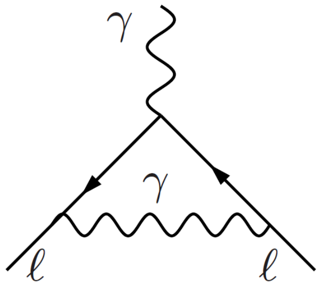
[d] Every Feynman diagram that is contained in figure 1 contributes.

3.1.1 Diagrams with virtual photons and muon loops

In these contributions only muons and photons are present. Thus they do not depend on muon mass.

- **1-loop diagram** [1 diagram]: The largest SM term, which contributes, was calculated by Schwinger in 1948. It is a single, famous one-loop (order-α) vertex correction diagram in figure 2 that exhibits

$$a_\mu^{(2)} = \frac{\alpha}{2\pi} \simeq 0.0011614. \quad ^{\mathrm{e}} \tag{14}$$



Slika 2. This is the one-loop Feynman diagram that contributes to the anomalous magnetic moment and corresponds to the first and largest quantum mechanical correction. Reproduced with permission of F. Jegerlehner (2007) [1].

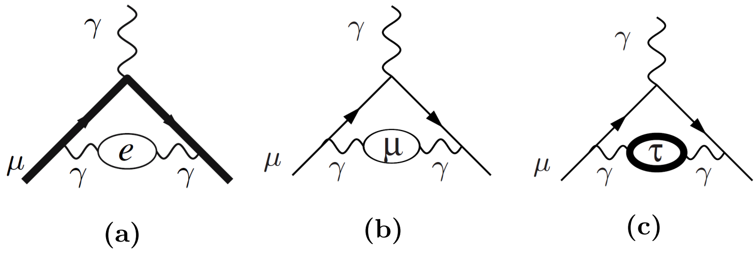
- **2-loop diagrams** [7 diagrams]
- **3-loop diagrams** [72 diagrams]: First it was calculated numerically, later they confirmed it analytically.
- **4-loop diagrams** [891 diagrams]
- **5-loop diagrams** [12672 diagrams]: For complete evaluation of 4 and 5-loop diagram one has to use numerical methods. These two contributions are updating with time, because of the progresses in computing technology [1, 5].

3.1.2 Mass dependent diagrams

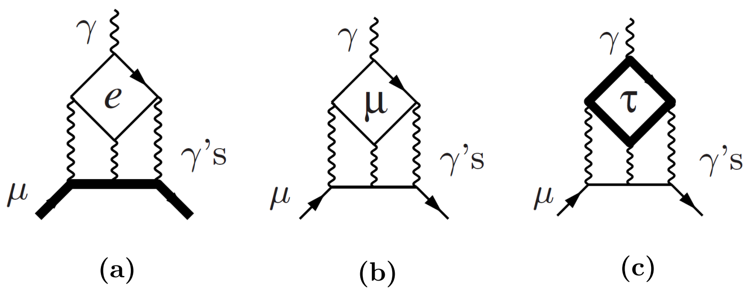
These terms include also electrons (lighter than muon) and tauons (heavier than muons). Hence they depend on masses specifically on mass ratios m_e/m_μ and m_τ/m_μ. There are two types of diagrams:

- **The photon vacuum polarisation (VP):** The lowest possible diagrams are 2-loop diagrams represented in figure 3. The diagram 3b, which has a muon loop, was already counted in the mass independent contribution. The higher the mass of a virtual lepton, the smaller the contribution to a_μ. Thus the diagram 3a yields the largest power correction of diagrams of this type.
- **Light-by-light scattering (LbL):** These diagrams are represented by closed fermion (in our case lepton) loops with four real photons attached ($\gamma\gamma \to \gamma\gamma$). One should know that closed fermion-loops with three photons vanish. Again, this contribution depends on the mass of the internal lepton, so that that the diagram that contributes the most is 4a, while the Feynman diagram 4b was actually considered in the first type of diagrams with virtual photons and muon loops [1, 5].

[e] We will express the perturbative order in powers of e, so that $a^{(n)}$ means an $\mathcal{O}(e^n)$ term. Note that the number of loops equals the power of α.



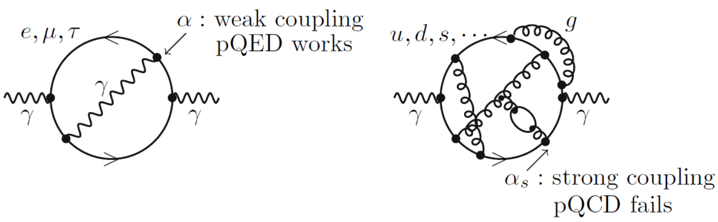
Slika 3. This figure shows the three different 2-loop QED VP diagrams. Reproduced with permission of F. Jegerlehner (2007) [1].



Slika 4. These are representative QED light-by-light scattering diagrams. Reproduced with permission of F. Jegerlehner (2007) [1].

3.2 QCD

An important correction to a_μ is to include hadrons. Quantum chromodynamics (QCD) is the gauge theory of strong interaction. In a_μ it shows up through the hadronic structure of the photon via vacuum polarisation starting at $\mathcal{O}(e^4)$ or light–by–light scattering starting at $\mathcal{O}(e^6)$. It looks like it is easy to derive, just replace lepton-loops by quark-loops. However, quarks are strongly interacting via gluons as described by the $SU(3)_{\text{color}}$ gauge theory, and while electromagnetic and weak interactions are weak in the sense that they allow us to perform perturbation expansions in the coupling constants, strong interactions are only weak at high energies. In the interesting regime, at energies below 2 GeV, the perturbative approach of QCD fails because the coupling constant increases (see figure 5).



Slika 5. One can see the main difference between QED and QCD: Because the coupling constant increases with decreasing energy, the quark-loop is full of quark–gluon plasma. This effect is the so-called anti–screening. Reproduced with permission of F. Jegerlehner (2007) [1].

Again, we divide the whole contribution into two parts:

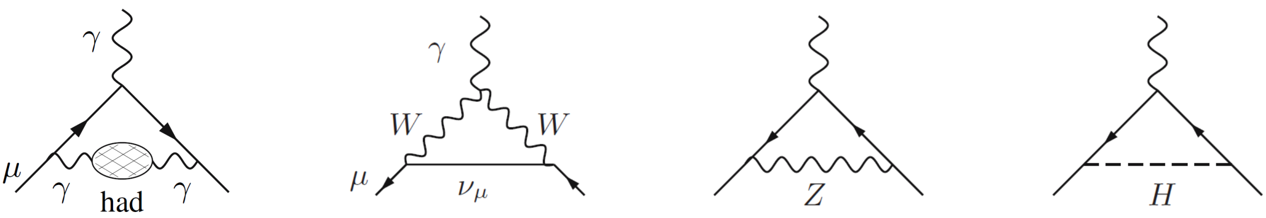
- **The photon vacuum polarisation (VP):** The leading hadronic impacts are VP type corrections, which can be evaluated by using causality, unitarity and low energy experimental data. The imaginary part of the photon self-energy Π_γ is calculated with the help of the optical theorem using the total cross-section of hadron production in e^-, e^+ annihilation. When we evaluate a dispersion integral containing Π_γ, we get the a_μ contribution. The largest contribution originates from the first Feynman diagram in figure 6.

- **Light-by-light scattering (LbL):** LbL scattering is a much more problematic set of hadronic corrections. Even for real–photon LbL scattering, perturbation theory is far from being able to describe reality, not to mention the non-perturbative approach [1, 5].

The total hadronic contribution dominates the uncertainty of a_μ^{SM}. Lattice QCD predictions are becoming better and will be crucial in providing good uncertainty estimates free from uncontrolled modelling assumptions [2].

3.3 Weak interaction

Weak interaction, described by the electroweak (EW) Standard Model, is the last type of interaction that is contributing. Muon interacts weakly via heavy spin-one gauge boson ($W^\pm$ or Z) or via a Higgs particle (H). The three leading weak contributions are shown in figure 6. The 2-loop diagrams were also taken into account, while the 3–loop effect has been estimated to be negligible for the accuracy needed at present. The weak interaction is due to suppressed factors and perturbatively done calculations under control [1, 5].



Slika 6. The first diagram is the most significant hadronic, while the next three are the most significant weak Feynman diagrams contributing to a_μ. The second diagram exhibits a non-Abelian triple gauge vertex so that its contribution provides a test of the Yang-Mills structure involved. Reproduced with permission of F. Jegerlehner (2007) [1].

Tabela 1. Summary table of the SM contributions to a_μ [2].

	value $[10^{-11}]$
QED	$116\,584\,718.95 \pm 0.08$
hadron VP	$6\,850.6 \pm 43$
hadron LbL	105 ± 26
EW	153.6 ± 1.0
a_μ^{SM}	$116\,591\,828 \pm 49$

4. Experiment

In this section, I will refer to two experiments: The Muon E821 Anomalous Magnetic Moment Measurement at Brookhaven National Laboratory (BNL) and The Muon $g-2$[f] experiment at Fermilab, also called Fermilab E989 experiment. a_μ was measured in three experiments at CERN (CERN I, CERN II and CERN III) and most recently in the E821 experiment. While CERN experiments were done in the sixties and seventies, and the experiment E821 measured a_μ five times between 1997 and 2001, the E989 first results will be released in 2018 [6].

[f] The anomalous magnetic moment is sometimes denoted as $g-2$.

If the muon with its magnetic moment moves in a magnetic field, its spin operator expectation value precesses around the magnetic field at a constant frequency known as Larmor frequency. Another kinematic effect of precession is if muons are moving at high velocities with a transverse acceleration. This is so-called Thomas precession. If high energetic muons are constrained to a circular orbit by a uniform magnetic field, as is the case in a storage ring[g], both Thomas and Larmor precessions are present. These muons get the following angular frequency

$$\boldsymbol{\omega}_{\mathrm{s}} = \frac{e}{m_\mu}\left[\left(a_\mu + \frac{1}{\gamma}\right)\mathbf{B} + \left(a_\mu + \frac{1}{\gamma+1}\right)\frac{\mathbf{E}\times\boldsymbol{\beta}}{c}\right], \tag{15}$$

where $\boldsymbol{\beta} = \mathbf{v}/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ is the relativistic Lorentz factor. In addition, muons also move in a circular orbit with a frequency known as the cyclotron frequency

$$\boldsymbol{\omega}_{\mathrm{c}} = \frac{e}{\gamma m_\mu}\left[\mathbf{B} + \frac{\gamma^2}{\gamma^2-1}\left(\frac{\mathbf{E}\times\boldsymbol{\beta}}{c}\right)\right]. \tag{16}$$

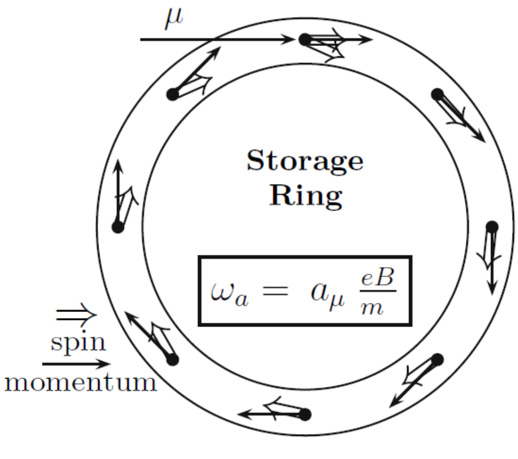
It is convenient to move to a reference frame that rotates with the velocity vector in order to keep the equations simple. Now the precision is given by the "anomalous" frequency which is the difference of angular frequencies $\boldsymbol{\omega}_{\mathrm{a}} = \boldsymbol{\omega}_{\mathrm{s}} - \boldsymbol{\omega}_{\mathrm{c}}$ [7],

$$\boldsymbol{\omega}_{\mathrm{a}} = \frac{e}{m_\mu}\left[a_\mu\mathbf{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\boldsymbol{\beta}\times\mathbf{E}}{c}\right]. \tag{17}$$

To simplify the equation relativistic muons must have a magic energy $E_{\mathrm{mag}} = \gamma_{\mathrm{mag}} m_\mu c^2 \sim 3.1\,\mathrm{GeV}$ for which $1/(\gamma_{\mathrm{mag}}^2 - 1) = a_\mu$. Luckily $\gamma_{\mathrm{mag}} = \sqrt{(1+a_\mu)/a_\mu} \simeq 29.378$ is large enough to provide the time dilatation factor for the unstable muons boosting the lifetime $\tau_\mu \simeq 2.197\cdot 10^{-6}\,\mathrm{s}$ to $\tau_{\mathrm{flight}} \simeq 6.454\cdot 10^{-5}\,\mathrm{s}$. γ_{mag} corresponds to $\beta \simeq 0.99942$. Muons with this velocity can be stored in a ring of reasonable size (with a diameter about 14 m). Now we are left with

$$\boldsymbol{\omega}_{\mathrm{a}} = \frac{e}{m_\mu} a_\mu \mathbf{B}, \tag{18}$$

which means that to measure a_μ, we must precisely measure ω_{a} and B (see figure 7) [1].



Slika 7. This is the spin precession in the storage ring plane.The precession amounts to 12 degrees per orbit. Not to scale. Adapted with permission of F. Jegerlehner (2007) [1].

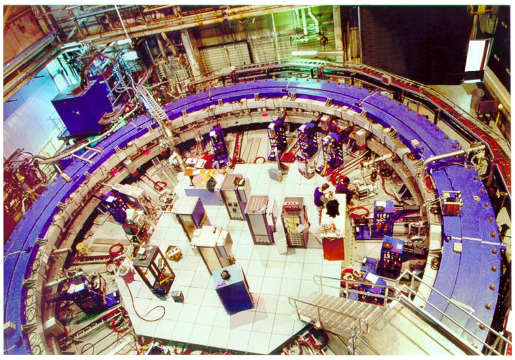
The CERN III, E821 and E989 experiments use the same principles. The Fermilab Muon $g-2$ experiment, the most improved of them all, is described in this section: A beam of muons with longitudinal polarisation and with magic energy E_{mag} is directed into a superconducting magnetic storage ring (see figures 8a and 8b) that has a very precisely known magnetic field [10].

Actually, the experiment begins with protons. About 12 times per second, Fermilab's accelerators smash a bunch of protons into a fixed target, creating different types of particles. Scientists are interested in the emerging pions, which quickly decay into muons with aligned spins. Magnets steer the pions and the resulting muons into a triangular-shaped tunnel called the Muon Delivery Ring.

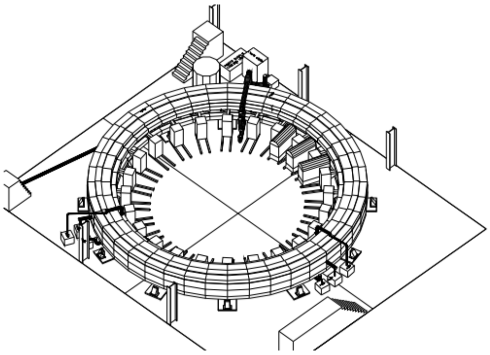
[g] A storage ring is a circular particle accelerator that maintains particles at the same energy.

As the particles travel hundreds of meters around the ring, essentially all of the pions decay into muons

$$\pi^+ \to \mu^+ + \nu_\mu, \qquad \pi^- \to \mu^- + \bar{\nu}_\mu. \tag{19}$$



(a) This picture shows the E821 storage ring. Reproduced with permission of E. Sichtermann [8].



(b) This is a sketch of BNL's storage ring. Reproduced with permission of E. Sichtermann (2006) [6].

Slika 8

This beam of muons is then transferred into the experiment's precision storage ring, which was used in the Brookhaven E821 experiment. The storage ring has been relocated to Fermilab in the summer of 2013 (see figure 9) [10].

As the muons travel around the ring, they are decaying into neutrinos and positrons[h]

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu, \qquad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu. \tag{20}$$

The neutrinos fly away undetected, but the positrons, which travel in the same direction that the muon's spin was pointing, can be measured [10]. The ω_a measurement is performed by detecting them and fitting the time distribution of the decays with a five-parameter fit. Positrons are detected using 24 calorimeters each composed of 54 PbF_2 Cherenkov crystals. Arising photons are detected by silicon photomultipliers and recorded using custom waveform digitizers. The calorimeters will be calibrated using a modern laser calibration system [3].

Slika 9. The Muon g-2 team successfully transported a 14-meters-wide electromagnet from Long Island, New York to the Chicago suburbs in one piece. The move took 35 days and traversed 5150 kilometres over land and sea. Reproduced from [9]. This image is in the public domain.

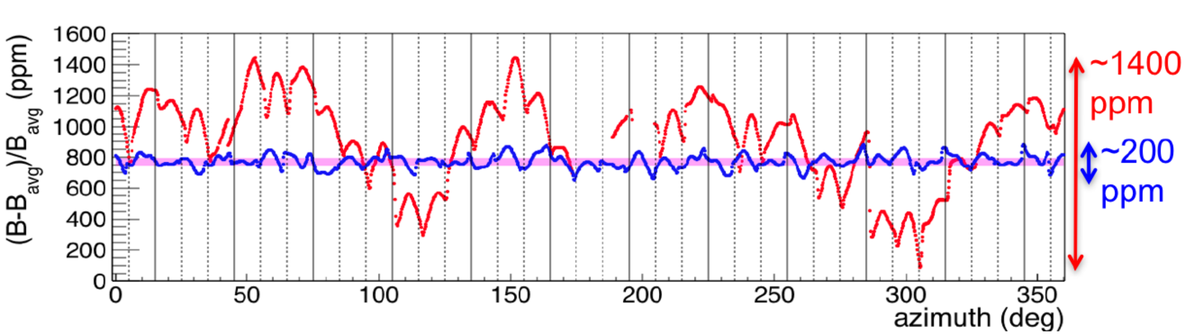
The measurement of the magnetic field is more precise if the precession frequency of the protons in the nuclear magnetic resonance (NMR) ω_p is measured as a proxy for B. The relationship between ω_a and B then changes from (18) to

$$a_\mu = \frac{\omega_\mathrm{a}/\omega_\mathrm{p}}{\mu_\mu/\mu_\mathrm{p} - \omega_\mathrm{a}/\omega_\mathrm{p}}, \tag{21}$$

where μ_μ and μ_p are muon's and proton's magnetic moment respectively. The magnitude of the magnetic field in the storage ring will be 1.45 T and must be as constant as possible. Fixed NMR

[h] Positron refers to both the electron if negative muon decayed and the positron if the muon was positive.

probes measure variations of the field during data taking. A trolley with mounted NMR probes periodically circles the interior of the ring to make precision measurements of the field in the muon storage region. It performs 6000 magnetic field measurements per trolley run. To complete the shimming of the magnet, which was already done, a special trolley outfitted with 25 NRM probes measured the field inside the ring while being tracked with a laser tracking system. The end result of shimming is shown in figure 10 [3].



Slika 10. Azimuthal variation in magnetic field. The red curve indicates $(B - B_{\text{avg}})/B_{\text{avg}}$ in October 2015, the blue in June 2016 and the pink band is the desired variation, which was achieved in August 2016. Reproduced with permission of W. Gohn (2016) [3].

5. Discussion

The SM theoretical summary is given in table 1. It should be compared to averaged experimental result of the E821 experiment

$$a_\mu^{\text{exp}} = (116\,592\,080 \pm 63) \cdot 10^{-11}. \tag{22}$$

The difference $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$ has a significance of 3.6 standard deviations [6]. What could be the reason for such a difference? Here are some options:

- **Statistical uncertainties:** All the experiments of this type are statistical processes, so physicists could have measured large statistical fluctuation which is far away from the theoretical value.

- **Underestimated systematic errors:** Many different measurements are done to get the final result. Maybe all these contributions to systematic uncertainties are underestimated.

- **Incomprehension of the Standard Model:** The current SM (renormalisable electroweak theory and QCD) may not have been correctly evaluated. No one doubts in the main terms derived from QED and the weak interaction, but the calculation of the hadronic part of a_μ is complex and in some parts approximate. We do not know precisely all the things that can contribute and that is the reason why the dispersive approach is used. Therefore, the biggest puzzle, in theory, is the hadronic part.

- **Physics beyond the Standard Model:** QED contributions have not an as important role for a_μ as for a_e. This is also the reason why a_μ is much more sensitive to BSM. There are contributions that cannot be explained with the SM, but we do not know what they actually are [4].

6. New physics

There are many interpretations of BSM that could handle with the discrepancy $a_\mu^{\exp} - a_\mu^{\mathrm{SM}}$:

- supersymmetry (SUSY) [11, 12]
- dark photon [11, 13]
- extra dimensions (ED) [14]
- additional Higgs bosons [15]
- radiative muon mass scenarios [16]
- anomalous W boson properties [16]
- new gauge bosons [16]
- leptoquarks [17]
- bileptons [16].

Rather than attempting to be inclusive, we concentrate briefly on two scenarios:

The first is new physics with supersymmetric particle loops (maybe the leading candidate explanation). Such a scenario could be quite real since generically supersymmetric models predict an additional contribution to a_μ^{SM}

$$a_\mu^{\mathrm{SUSY}} \simeq \mathrm{sign}(\mu) \cdot 130 \cdot 10^{-11} \cdot \left(\frac{100\,\mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta, \tag{23}$$

where m_{SUSY} is a representative supersymmetric mass scale, $\tan\beta \simeq 3 - 40$ is a potential enhancement factor and $\mathrm{sign}(\mu) = \pm 1$. SUSY particles in the mass range $100 - 500\,\mathrm{GeV}$ could be the source of the deviation $a_\mu^{\exp} - a_\mu^{\mathrm{SM}}$. If this is the case, those particles should be observed at the Large Hadron Collider (LHC) at CERN [11].

A recent popular scenario involves the "dark photon", a relatively light hypothetical vector boson from the dark matter sector that couples to SM of particle physics through mixing with the ordinary photon. Physicists predict that it couples to ordinary charged particles with strength $\varepsilon \cdot e$ and produces an additional muon anomalous magnetic moment contribution

$$a_\mu^{\text{dark photon}} = \frac{\alpha}{2\pi}\varepsilon^2 F(m_{\mathrm{V}}/m_\mu), \tag{24}$$

where

$$F(x) = \int_0^1 \frac{2z(1-z)^2}{(1-z)^2 + x^2 z}\,\mathrm{d}z. \tag{25}$$

For values of $\varepsilon \sim 1 - 2 \cdot 10^{-3}$ and $m_{\mathrm{V}} \sim 10 - 100\,\mathrm{MeV}$, the dark photon, which was originally motivated by cosmology, can provide an achievable solution to the a_μ discrepancy. Searches for the dark photon in that mass range are currently underway at Jefferson Lab, USA, and MAMI in Mainz, Germany [11].

7. Conclusion

The a_μ result from BNL cannot at present be explained by the established theory. The Fermilab Muon $g-2$ team wants to make the measurement even better. They have a plan to reduce the total uncertainty by a factor of four. If this happens and they measure the same value as (22), the difference between theory and experiment could become $7.5\,\sigma$ [3]. This could be a strong hint of physics beyond the Standard Model. In this case, supersymmetry can explain the data, but we would need other experiments to show that the postulated particles can exist in the real world [4].

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