SPACE-TIME RIPPLES

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Recent first direct detection of gravitational waves opened a new window on the universe. Gravitational waves arise from small perturbations of space-time and from their characteristics it is possible to determine properties of astrophysical objects from which they originate. In this seminar Einstein equation is introduced to describe gravitational waves and ascertain their properties. Furthermore, possible astrophysical sources are presented, as well as the first directly and indirectly detected event.

VALOVANJE PROSTORA-ČASA

Nedavna prva neposredna detekcija gravitacijskih valov je odprla novo okno v vesolje. Gravitacijski valovi nastanejo iz majhnih motenj prostora-časa in iz njihovih značilnosti lahko določimo lastnosti astrofizikalnih teles iz katerih izvirajo. V članku je predstavljena Einsteinova enačba, ki omogoča opis gravitacijskih valov in njihovih lastnosti. Predstavljeni so tudi možni astrofizikalni izvori ter prva posredno in neposredno zaznana dogodka.

1. Introduction

Gravity is a natural occurrence, which affects the very fabric of space-time. Isaac Newton described it as a force, which acts between bodies with mass, but this theory becomes inaccurate in the presence of strong gravitational field. In such systems not only the Newton's approach gives vague results, there are also phenomena, which are not predicted, for instance, precession of Mercury's perihelion and gravitational waves. However, in 1915 Albert Einstein established a theory, which presented gravity in a new way and led to many unique discoveries.

Einstein's general theory of relativity describes gravity as a curvature of space-time, a mathematical model that links space and time components. His theory predicts, that an object with mass/energy curves space-time and higher mass results in stronger curvature. Furthermore, if such an object moves through space, the curvatures moves with it. However, the reaction is not instantaneous, since the information can only propagate at the speed of light. In some cases, which we will discuss below, the motion causes ripples in space-time, which are called *gravitational waves* (GW).

For an entire century it was uncertain, whether GW exist, until in 2015 Laser Interferometer Gravitational-wave Observatory (LIGO) detected them directly for the first time, which marked the beginning of a new era in astrophysics.

2. General relativity

Gravitational waves are caused by perturbation of space-time and in order to discuss them, it is necessary to derive Einstein equation. In general relativity space-time is represented by a manifold[1] with metric, locally treated as a four dimensional tangent plane $\mathbb{R}^4$.

2.1 Metric

Metric is a function necessary to define properties of space-time , such as distance between two points, given by four-vectors x^μ and x^ν. For instance, flat space-time is described with Minkowski metric

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[1] Topological space that can be approximated as N dimensional Euclidean space near each point.

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{1}$$

Distance does not change even if we swap vectors (or indices) and if one of them equals zero, so does their scalar product. Therefore, metric tensor is bilinear, non-degenerate and symmetric.

2.2 Christoffel symbols

In a flat space-time, tangent plane is determined by tangent vectors, calculated with partial derivatives along coordinates. However, as stated before, gravity affects space-time in a way, which causes it to curve. Tangent vector $\vec{v}$ is no longer represented with partial, but with covariant derivative $\vec{\nabla}_u\vec{v}$, a derivative along vector $\vec{u}$, expressed in the following way[1]

$$\nabla_\beta v^\alpha = \partial_\beta v^\alpha + \Gamma^\alpha_{\mu\nu}v^\mu. \tag{2}$$

$\Gamma^\alpha_{\mu\nu}$ are Christoffel symbols, a connection between two tangent spaces, defined as

$$\Gamma^\beta_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left(\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\alpha\mu} - \partial_\alpha g_{\mu\nu}\right). \tag{3}$$

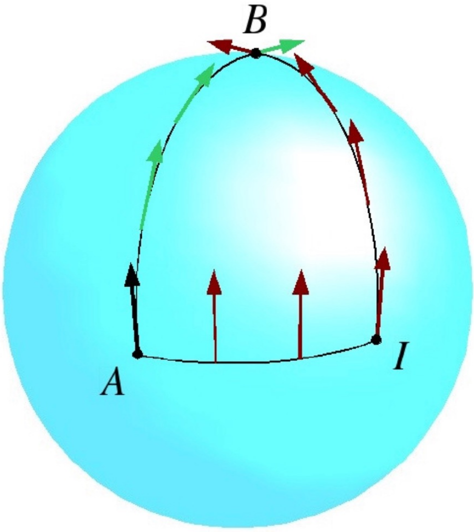
In essence, they measure deviation of parallely transported coordinate vector field. Moreover, symbols must be symmetric ($\Gamma^\beta_{\mu\nu} = \Gamma^\beta_{\nu\mu}$) and are not tensors.

2.3 Einstein equation

Effect of curvature can be interpeted from a parallel transport of a vector along two different paths. Figure 1 illustrates such operation and it can be noticed that vectors differ at the end point.This deviation is revealed by Riemann tensor

$$R^\alpha_{\beta\mu\nu} = \partial_\mu\Gamma^\alpha_{\beta\nu} - \partial_\nu\Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\sigma\mu}\Gamma^\sigma_{\beta\nu} - \Gamma^\sigma_{\beta\mu}\Gamma^\alpha_{\sigma\nu}. \tag{4}$$

a 4×4 matrix with many symmetries ($R^\alpha_{\beta\mu\nu} = R^\alpha_{\beta\nu\mu}$, $R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu}$, etc.).



Slika 1. Transportation of two identical vectors starting in point A along different paths in curved space, leads to two different vectors at point B[2].

In order to derive Einstein equation, Ricci tensor and scalar are necessary. Ricci tensor is introduced by contracting Riemann tensor $R_{\alpha\beta} = g^{\mu\sigma}R_{\alpha\mu\beta\sigma}$ and expresses evolution of a parallely transported small volume in space. Moreover, Ricci scalar is a further contraction, defined as $R = g^{\mu\nu} R_{\mu\nu}$, which measures deviation between areas of a flat and curved sphere.

Eistein equation is a set of equations, connecting curvature and matter, which follows from Poisson equation for Newtonian potential: $\nabla^2 \Phi = 4\pi G \rho$. Matter is represented by stress energy tensor $T_{\mu\nu}$, which expresses, how particles affect space - it describes momentum, flux of energy, as well as density in space-time. Moreover, second derivative of gravitational potential Φ implies a relation to second derivative of metric tensor - Riemann tensor. By taking into consideration conservation of energy, Newton's limit for Φ and symmetry conditions, Einstein equation is introduced[3]

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{5}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is Einstein tensor. Λ is cosmological constant, a parameter which includes accelerated expansion of the universe in the equation, but does not play important role at the distances of currently detectable astrophysical sources of GW. Therefore, it is assumed $\Lambda = 0$.

3. Gravitational waves

GW are ripples in space-time. The question is, if they can in fact be described with wave equation, what their polarization, flux and power are.

3.1 Wave equation and polarization

Region, distant from the source of GW, can be described with Minkowski metric and GW as perturbation, introduced as a small correction ($\| h_{\mu\nu} \| \ll 1$)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{6}$$

Next step is to insert perturbed metric into Christoffel symbols, to derive Einstein tensor. What is more, a trace of perturbation is defined as $h = \eta^{\mu\nu} h_{\mu\nu}$, as well as a traceless perturbation tensor $\overline{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$.

In general relativity physical laws do not change, if the coordinates are transformed, and by choosing correct gauge (coordinate condition), problems become easier to solve. Therefore, Lorentz gauge condition $\partial^\nu \overline{h}_{\mu\nu} = 0$ is applied (similar to electromagnetism - EM). Moreover, slightly perturbed space-time can be described with the approximation of weak gravitational field. In this limit all higher derivative orders are neglected and Einstein equation is expressed as[3]

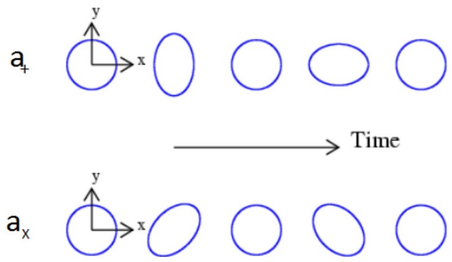
$$\Box \overline{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \tag{7}$$

where $\Box = \eta^{\sigma\rho} \partial_\rho \partial_\sigma = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2$. Equation (7) is analogous to the wave equation in EM $\Box A_\mu = -\mu_0 J_\mu$. A_μ contains information about vector and scalar fields, while J_μ represents a vector incorporating dependency on electric charge and current density.

Furthermore, in vacuum ($T_{\mu\nu} = 0$) simple exponential solutions can be derived

$$\overline{h}_{\mu\nu} = a_{\mu\nu} e^{i(kz - \omega t)}. \tag{8}$$

In this case propagation with constant wave vector k in z direction is considered, whereas $a_{\mu\nu}$ carries information about polarization and amplitude of waves. Initially $a_{\mu\nu}$ has 10 independent components from symmetry condition. However, by assuming TT gauge, given by transverse $a_{\mu\nu} k^\mu = 0$ and traceless condition as $a_{\mu\nu} \eta^{\mu\nu} = 0$, only two possible polarizations remain: $+$ and $\times$. The first one stretches and squeezes distances between particles periodically in horizontal and vertical directions. Moreover, the other has the same effect only shifted by 45° (shown in Figure 2).



Slika 2. Effect of a "+"and "×"polarized GW, coming from z direction, on a ring of particles[4].

3.2 Energy flux and luminosity

Important properties of any radiation are energy flux and luminosity. Energy flux $\mathcal{F}$ expresses the rate of energy, that is transported through a certain surface and is directly related to stress energy tensor $T_{0z}c = \mathcal{F}$. To obtain it, it is required to average previously neglected higher derivative orders over wavelenghts, as well as apply TT gauge. Moreover, in the case of simple plane waves with constant wave vector k_μ, an illustrative result, with three non zero components is introduced[5]

$$T_{00} = \frac{T_{zz}}{c^2} = -\frac{T_{0z}}{c} = \frac{c^2}{32\pi G}\omega^2(a_+^2 + a_\times^2), \tag{9}$$

which gives an estimate for energy flux of GW with frequency f in the z direction and strain h [2]

$$\mathcal{F} \sim 0,3\left(\frac{f}{1\text{kHz}}\right)^2\left(\frac{h}{10^{-21}}\right)^2\frac{\text{W}}{\text{m}^2}. \tag{10}$$

By considering relation between energy radiated in GW and flux: $\Delta E = \epsilon Mc^2 = 4\pi r^2\mathcal{F}\tau$, where ϵ is efficiency and τ duration of radiation, it is possible to derive an estimate for strain in a system at a distance r from the observer

$$h \sim 2\cdot 10^{-19}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{\text{Mpc}}{r}\right)\left(\frac{\text{kHz}}{f}\right)\left(\frac{\text{ms}}{\tau}\right)^{1/2}\epsilon^{1/2}. \tag{11}$$

Therefore, astrophysical systems need to have large masses and high frequencies in order to inflict measurable strain on the observer located on the Earth.

But what causes gravitational waves? As we know, GW behave very similar to EM waves, radiation produced due to dipole radiation. This gives reason to look at different multipoles also in the case of gravitational radiation. It is ascertained that there is no monopole radiation, as a result of mass conservation, and no dipole radiation, as long as angular momentum is conserved[5]. Therefore, quadrupole radiation is considered, expressed with quadrupole moment Q_{ij}.

Analogously to EM luminosity $L_{EM} = \frac{1}{3}\frac{\mu_0}{4\pi} < \ddot{D}_i\ddot{D}^i >$, where D_i represents dipole moment, power radiated in the form of GW is $L = \frac{1}{5}\frac{G}{c^5} < \dddot{Q}_{ij}\dddot{Q}^{ij} >$[2]. The difference in the derivative order arises from replacement of four vectors with monopoles: four current J_μ can be replaced with first time derivative of dipole moment, while stress energy tensor is expressed with the second time derivative of quadrupole moment. For a system with mass M, radial size R and orbital frequency ω quadrupole moment can be approximated as $Q \sim sMR^2$ and its third derivative like $\dddot{Q} \sim s\omega^3MR^2$. Therefore, a more representative assessement can be written as:

$$L \sim \frac{G}{c^5}s^2\omega^6M^2R^4 \sim \frac{c^5}{G}s^2\left(\frac{v}{c}\right)^6\left(\frac{R_\text{S}}{R}\right)^2. \tag{12}$$

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[2] Strain is relative change of length ($h = \frac{\delta L}{L}$), induced by stretching and squeezing, as a result of nature of gravitational waves. It is also amplitude of perturbation tensor.

Factor $s \in [0,1]$ serves as a measure of deviation from spherical symmetry of an astrophysical system, v is its orbital velocity and $R_\mathrm{S} = \frac{2GM}{c^2}$ is Schwarzschild radius. Furthermore, GW sources with high velocities, high asymmetries and small sizes produce more radiation, which gives a general direction in the search of known astrophysical objects, that can exact measurable strain.

4. Sources

There are several types of astrophysical sources, which may produce GW. In essence, that would mean asymmetric mass distribution, which must be accelerated and a few examples would be compact binaries, supernovae, neutron stars, etc. These events are rare and the probability of them occurring in our galaxy is very low. Therefore, bodies which emit GW that LIGO can measure, are all expected to be located in other galaxies.

4.1 Bursts

Short bursts of GW are expected to be related to transient events, such as supernovae or starquakes on magnetars. Massive star ($M \gtrsim 8M_\odot$) ends its life as a supernova - a very energetic ejection of a stellar envelope. Many of the observed remaining cores showed high velocities, which led to the explanation, that the envelope's mass was ejected asymmetrically. Therefore, these explosions could generate GW, as well as produce necessary "kick". For example, a supernova emitting GW with frequency 1 kHz, which forms a 10 $\mathrm{M}_\odot$ black hole 15 Mpc away, would induce strain of $h \sim 10^{-22}$. Radiated flux would be $\mathcal{F} \sim 0.003$ Wm^{-2} and luminosity $L \sim 10^{46}$ W[5].

During the collapse of a stellar core magnetic flux is conserved, which could lead to neutron stars with high magnetic fields ($10^4 - 10^8$ T). However, observations revealed neutron stars with even higher B ($10^8 - 10^{11}$ T), which were named magnetars. It is accepted that in right conditions, magnetic field can be further amplified by conversion of heat and rotational energy[6]. Repeating gamma ray signals and anomalous X-ray pulses were detected from such objects, which is why many believe that magnetars could also generate GW. For instance, they could be produced by cracks in magnetar's crust and if it were located about 10 kpc away with $B \sim 10^{11}$ T, they should inflict strain $h \sim 10^{-27}$. Furthermore, luminosity would be $L \lesssim 10^{33}$ W and flux $\mathcal{F} \lesssim 10^{-11}$ Wm^{-2}[7]. These events would be very hard to detect with LIGO's current equipment.

4.2 Continuous

Continuous GW could originate from a spinning, asymmetrical massive body. GW produced in such a way, have almost constant frequency and amplitude (hence continuous). One example would be a spinning neutron star, where any imperfection in spherical distribution of mass could lead to gravitational radiation. Imperfections could arise from missaligned magnetic fields with rotatational axis, accretion, distortions in the crust etc. Furthermore, rotational energy of the star is decreased, as a result of energy radiated away. Continuous GW, generated by neutron stars, are very weak and cause strain $h \approx 10^{-25}$, which makes them hard to detect. By accumulating observational data for several months, it should be possible to get higher strain.

Additional candidates are supermassive black hole (SMBH) binaries. SMBH are black holes with masses in the range of $10^6 - 10^{10}$ $\mathrm{M}_\odot$ and are located in many (if not in every) center of the galaxy. In a collision of two galaxies it may occur, that such objects form a binary system. This is similar to compact binaries, except that it takes them even longer to merge, while they produce practically continuous signal and stronger than in compact binaries due to higher masses. GW from SMBH binaries could be measured by space interferometers.

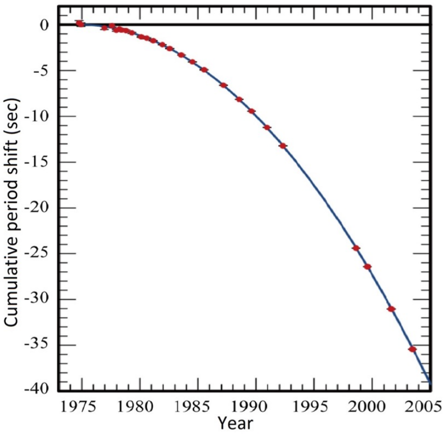
4.3 Compact binary systems

Compact objects are the final form of stars at the end of their life. These are white dwarfs (WD), neutron stars (NS) or black holes (BH). In compact binary systems two compact bodies orbit around the mutual center of mass. Due to the energy loss through gravitational waves, orbits are first circularized. In addition, bodies begin to move closer and start to orbit faster, until they finally merge. From the equation (12) it is inferred, that as the orbital velocity increases, while distance between objects decreases, luminosity of gravitational radiation increases. This process produces a signal, which rises rapidly in frequency, also known as chirp.

However, coalescence time depends on the mass of the system, as well as initial separation and in general, it could take several billion years for the bodies to merge. A rough estimate for a number of mergers in a galaxy similar to ours is: $5 \cdot 10^{-9}$/Myr for a BH-BH binary, $5 \cdot 10^{-8}$/Myr for NS-BH pair and for NS-NS 10^{-8}/Myr[4]. Moreover, LIGO can measure only GW produced in the final few seconds before objects collide. This makes such events even more rare and hard to detect. For instance, two neutron stars with $M = 1.4\ \mathrm{M}_\odot$, 50 billion light years away, separated by 20 km and orbital frequency 400 Hz (consequently GW frequency is 800 Hz), would produce strain $h \sim 10^{-21}$. Such signal should be measurable by LIGO. Energy flux detected on Earth would be $\mathcal{F} \sim 0.1\ \mathrm{Wm}^{-2}$ and radiated power $L \sim 10^{48}$ W[4].

4.3.1 Hulse-Taylor system

Compact binary systems are the first systems, from which GW were confirmed directly and indirectly. Indirect evidence came from the PSR 1913+16, a neutron star binary (one of them is a pulsar), discovered by R. A. Hulse and J. H. Taylor, Jr. in 1974. The two scientists noticed a periodical radio signal and continued to measure it for several years. From different arrival times of pulses, they deduced the size of the orbit and they realized that it was shrinking. By comparing these results to the ones predicted by energy loss through GW, they concluded that the reason must be gravitational radiation (Figure 3).

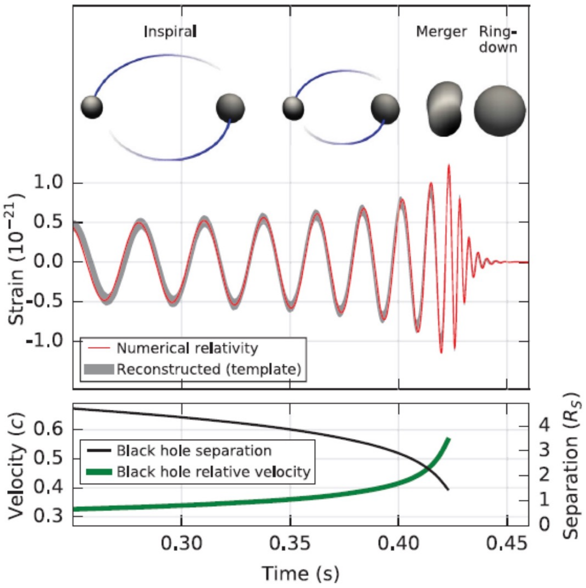


Slika 3. Change of period over several years of observation of Hulse-Taylor system compared to the model given by general theory of relativity[8].

4.3.2 GW150914

Direct confirmation came from GW150914 event, which was detected by LIGO in Houston (H1) and in Lousiana (L1). These gravitational wave detectors work on a principle of laser interferometry, similar to Michelson interferometer and they observed first GW on September 14th 2015. LIGO detected an oscillation, which showed chirp waveform, followed by chaotic behaviour, which quickly diminished. L1 interferometer detected signal 7 ms before the H1 detector, indicating the general direction of the source of the waves. Unfortunately position of the source could not be measured with a better precision, because VIRGO (GW observatory in Italy), as well as GEO600 (GW observatory in Germany) were not operational at that time.

Calculations from observation revealed large common mass and high orbital frequency implying that the system was a black hole binary. Figure 4 shows reconstructed signal from H1 in comparison to estimated strain from the GW150914. It also illustrates, how the separation and velocity changed with time, until the merger. When the two bodies finally merged, a new highly deformed black hole was formed. Due to deviation from spherical symmetry, GW were released, producing a system without any distortion. This is known as ringdown phase.



Slika 4. Evaluated strain, Keplerian effective separation in units of Schwarzschild radius and effective velocity in the system GW150914[9].

On October 12th and December 26th 2015 LIGO detected signals, which both most likely came from a black hole binary. However, the first one is regarded only as a possible candidate, due to weak signal strength. Characteristics of all three events are shown in Table 1.

Properties Event	Primary mass [$M_\odot$]	Secondary mass [$M_\odot$]	Final mass [$M_\odot$]	Radiated energy [$M_\odot c^2$]	Distance [Mpc]
GW150914	$36.2^{+5.2}_{-3.8}$	$29.1^{+3.7}_{-4.4}$	$62.3^{+3.7}_{-3.1}$	$3.0^{+0.5}_{-0.4}$	420^{+150}_{-180}
LVT151012	23^{+18}_{-6}	13^{+4}_{-5}	37^{+13}_{-4}	$1.5^{+0.3}_{-0.4}$	440^{+150}_{-180}
GW151226	$14.2^{+8.3}_{-3.7}$	$7.5^{+2.3}_{-2.3}$	$21.8^{+5.9}_{-1.7}$	$1.0^{+0.1}_{-0.2}$	1000^{+100}_{-100}

Tabela 1. Characteristics of three events detected by LIGO[10].

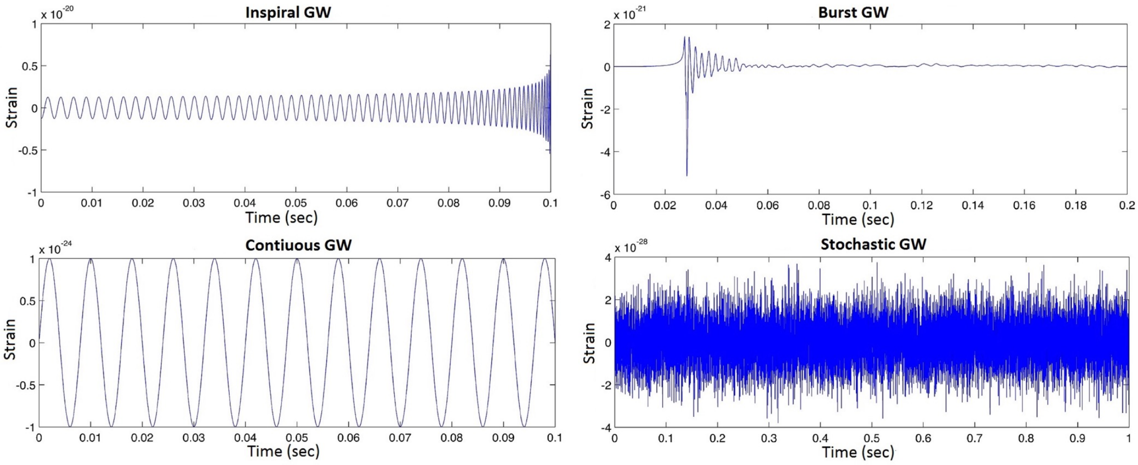
4.4 Other

Gravitational waves are also a possible way to study the universe right after the Big Bang ($\approx 10^{-36}$-10^{-32} s)[11]. As they couple weakly with matter, they could procure information even from time way before cosmic microwave background (CMB), a radiation produced at the time of recombination of protons and electrons[5]. This short period of time is known as inflation - a theoretical event of exponential expansion of space. If it really took place, it should have released GW, which affected polarization of CMB. These changes in CMB can be measured and could help deduce amount of energy released during inflation, as well as determine more accurately, when it happened. Released waves should be isotropic and stochastic. Furthermore, they would be highly red-shifted, due to the expansion of the universe.

Moreover, gravitational radiation should also be induced in extreme mass ratio inspirals, which occur when a compact object, which orbits around a SMBH, looses energy due to GW. This results in a slow inspiral, which makes possible observation of GW emitted over longer time scales. Such event would help in understanding space-time of SMBH, as well as stellar systems near galaxy centers.

Additional source could arise due to a star that gets disrupted and its gas starts to fall on SMBH or BH. Gas infall causes mass distribution of a compact object to deviate from spherical symmetry. Therefore, black hole begins to "ring"and produce GW.

In Figure 5 it is demonstrated, that these different astrophysical sources produce different strain patterns. Furthermore, it is also noticed, that each source has its typical frequency range.



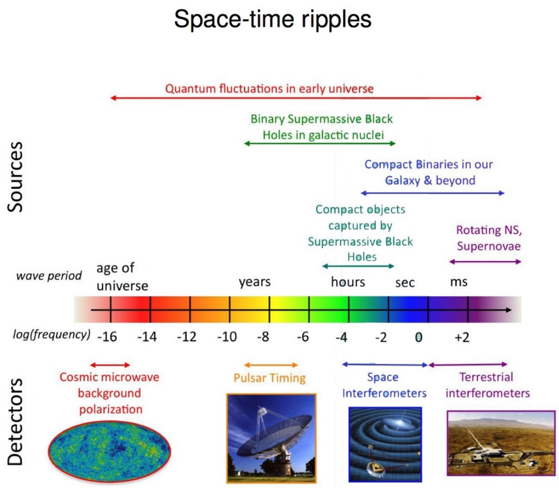
Slika 5. Comparison of signal shape, generated in different astrophysical systems[11].

4.5 Gravitational waves spectrum

In a similar manner as EM waves also GW have a spectrum, which is shown in Figure 6 together with different way of detecting them. Waves with shortest frequencies produced in the early stages of the universe, could be detected with Planck satellite, by measuring the polarization of CMB.

Periods of the order of years correspond to binary SMBHs, which could be detected with pulsar arrays or space interferometers such as Laser Interferometer Space Antenna. LISA is a project to build an interferometer for GW detection in the Solar orbit and is currently scheduled for launch in 2034.

GW with frequencies from 0.01 Hz, can be measured with ground laser interferometers (LIGO). These GW are produced by compact binary systems, rotating NS or supernovae.



Slika 6. The gravitational wave spectrum[12].

5. Conclusion

In accelerated systems, where quadrupole moment is not conserved, gravitational waves are generated. These waves travel through the universe with velocity c, as an undulation of space-time. Due to space-time's stiffness, only GW produced in catastrophic events, are measurable with current technology.

GW allow scientists to observe objects, that do not emit electromagnetic waves. Furthermore, they interact very weakly with matter, which provides uncontaminated information and knowledge about events, undetectable by telescopes.

Observation of GW events, was made possible with a large interferometer LIGO. During its first run, which lasted from September 2015 to January 2016, LIGO detected two sources and one potential candidate. Since then it underwent some modifications to improve its sensitivity and began second observing run in November 2016. Therefore, it is expected, that even more detections will be made soon. In addition, an ongoing project LISA could observe GW originating from different systems.

Direct detection brought many important revelations. It confirmed Einstein's interpretation of gravity, as well as proved existence of binary black holes. Moreover, BH with masses about 30 $M_\odot$ were observed for the first time. Further detections and analysis of gravitational waves could provide more information about NS and their equation of state. It could also contribute to understanding last stages of stellar lives - mechanisms of supernovae and formation of compact bodies. In essence, analysis of GW offers a complementary view on the astrophysical phenomena and it could play an important part in acquiring a more detailed picture of the universe.

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